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DESIGN ON THE FREQUENCY PARAMETER PLANE
WITH
APPLICATION TO ROLL STABILIZATION OF SHIP

Manop Vachanaratana

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THESIS

Design on the Frequency Parameter Plane
with
Application to Roll Stabilization of Ship

by

Manop Vachanaratana

Thesis Advisor:

G. J. Thaler

December 1973

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Design on the Frequency Parameter Plane
with
Application to Roll Stabilization of Ship

by

Manop Yachanaratana
Lieutenant Junior Grade, Royal Thai Navy
B.S., Royal Thai Naval Academy, 1964

Submitted in partial fulfillment of the
requirements for the degree of

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NAVAL POSTGRADUATE SCHOOL
December 1973

ABSTRACT

Frequency parameter plane techniques are developed for the analysis and synthesis of both linear and nonlinear systems. The transmission function of a dynamic system contains two parameters which are adjusted to provide desired performance. These parameters represent elements of the system.

The four dimensional parameter space relating magnitude, frequency, and the two parameters may be represented by seven two dimensional projections of the parametric curves. Computer programs are developed for the computation, and graphs of results are obtained. Interpretations of the graphs can be used for design of systems such as tank stabilizers.

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I. INTRODUCTION

The analysis and synthesis of control systems may be accomplished using the frequency parameter plane technique. A primary consideration in control system design is the adjustment of the system frequency response. To achieve this, the frequency parameter plane technique may be employed and is based on a graphical procedure for considering four parameters, alpha, beta, frequency and magnitude of the transmission functions. The objective is to choose values for alpha and beta (or functional relationships if the design is to be nonlinear) to provide the desired frequency response characteristics. This choice is possible through proper use of the frequency parameter plane curve families.

The goal of this thesis is design using analysis techniques. The parameter frequency response method is used to synthesis and design systems. A steam-engine governor is used in the analysis as an illustration. The antiroll tank stabilizer of a ship is also designed using the frequency parameter plane, for both passive and active tanks. The tank size and active device characteristics are represented by the parameters, which are varied. Parameter plane curve families are used in the design procedure presented in later chapters.

II. PARAMETER PLANE ANALYSIS

Dynamic systems may be either linear or nonlinear. Parameter plane analysis permits us to study dynamic behavior by treating system parameters as variables. One first determines the characteristic polynomial of the system which may be in the form

$$F(s) = \sum_{k=0}^N a_k s^k = a_N s^N + a_{N-1} s^{N-1} + \dots + a_1 s + a_0 = 0 \quad (2.1)$$

where a_k = coefficients which are functions of the parameters.

$$s = \sigma + j\omega = -\zeta\omega_n + j\omega_n \sqrt{1-\zeta^2} = \omega_n e^{j\theta}$$

By substituting the value of s into equation 2.1, equation 2.2 is obtained.

$$F(s) = \sum_{k=0}^N a_k \omega_n^k e^{jk\theta} \quad (2.2)$$

Since equation 2.2 has complex coefficients it can be converted into two equations by requiring reals and imaginaries to be zero independently.

$$\begin{aligned} \text{Note that } s^k &= \omega_n^k e^{jk\theta} = \omega_n^k (\cos k\theta + j \sin k\theta) \\ &= X_k + j Y_k \end{aligned}$$

$$\text{where } X_k = \omega_n^k \cos k\theta = \omega_n^k \cos(k \cos^{-1} \zeta) = (-1)^k \omega_n^k T_k(\zeta)$$

$$Y_k = \omega_n^k \sin k\theta = \omega_n^k \sin(k \cos^{-1} \zeta) = (-1)^k \omega_n^k U_k(\zeta)$$

$T_k(\zeta)$ and $U_k(\zeta)$ are Chebyshev polynomials with values as shown in Table 1, 2, and 3.

It follows that

$$\begin{aligned} \sum_{k=0}^N a_k [(-1)^k \omega^k T_k(\zeta)] &= 0 \\ \sum_{k=0}^N a_k [(-1)^k \omega^k U_k(\zeta)] &= 0 \end{aligned} \quad (2.3)$$

are simultaneous equations which may be solved for two undefined parameters in the a_k .

For many practical problems the algebraic form of the coefficients is one of the following two cases:

1. $a_k = b_k \alpha + c_k \beta + d_k$
2. $a_k = b_k \alpha + c_k \beta + h_k \alpha \beta + d_k$

Case 1 can be solved by Cramer's rule and the results for α and β are

$$\begin{aligned} \alpha &= \frac{C_1 D_2 - C_2 D_1}{B_1 C_2 - B_2 C_1} \\ \beta &= \frac{B_2 D_1 - D_2 B_1}{B_1 C_2 - B_2 C_1} \end{aligned}$$

where

$$B_1 = \sum_{k=0}^N (-1)^k b_k \omega_n^k T_k(\zeta); \quad B_2 = \sum_{k=0}^N (-1)^k b_k \omega_n^k U_k(\zeta)$$

$$C_1 = \sum_{k=0}^N (-1)^k c_k \omega_n^k T_k(\zeta); \quad C_2 = \sum_{k=0}^N (-1)^k c_k \omega_n^k U_k(\zeta)$$

$$D_1 = \sum_{k=0}^N (-1)^k d_k \omega_n^k T_k(\zeta); \quad D_2 = \sum_{k=0}^N (-1)^k d_k \omega_n^k U_k(\zeta)$$

Case 2 by substituting equation 2.3 rearranges into the form.

$$\alpha B_1 + \beta C_1 + \alpha\beta H_1 + D_1 = 0$$

where the definitions of case 1 still apply, and

$$H_1 = \sum_{k=0}^N (-1)^k h_k \omega_n^k T_k(\zeta); \quad H_2 = \sum_{k=0}^N (-1)^k h_k \omega_n^k U_k(\zeta)$$

Eliminating β the solutions are obtained

and corresponding values for β are

$$\beta_{1,2} = -\frac{B_1 \alpha_{1,2} + D_1}{H_1 \alpha_{1,2} + C_1} = -\frac{B_2 \alpha_{1,2} + D_1}{H_2 \alpha_{1,2} + C_2} \quad \dots \dots \dots \quad (2.5b)$$

Eliminating α , the following solution for β

$$\beta_{1,2} = \frac{-f \pm \sqrt{f^2 - 4bd}}{2b} \quad \dots \dots \dots \quad (2.6a)$$

and the corresponding values for α are

$$\alpha_{1,2} = - \frac{C_1 \beta_{1,2} + D_1}{H_1 \beta_{1,2} + B_1} = - \frac{C_2 \beta_{1,2} + D_2}{H_2 \beta_{1,2} + B_2} \quad \dots \dots \dots \quad (2.6b)$$

where the notation is the following

$$a = B_2 H_1 - B_1 H_2; \quad c = C_1 D_2 - C_2 D_1$$

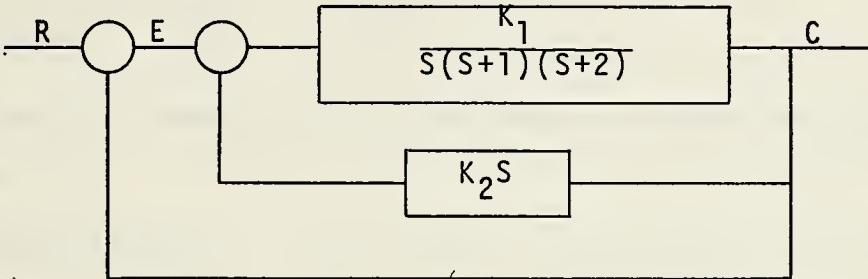
$$b = C_2 H_1 - C_1 H_2; \quad d = B_1 D_2 - B_2 D_1$$

$$e = B_2 C_1 - B_1 C_2 + H_1 D_2 - H_2 D_1$$

$$f = C_2 B_1 - B_2 C_1 + H_1 D_2 - H_2 D_1$$

As can be seen, there are generally two pairs of solutions (α, β) in terms of ω_n and ζ . The solutions are given by equations 2.5a and 2.5b provided $a \neq 0$. If $a = 0$ the solutions are given by equations 2.6a and 2.6b provided $b \neq 0$. When $a = b = 0$ case 2 reduces to case 1.

Illustrations I. A feedback control system is as shown



The closed-loop transfer function is:

$$T(s) = \frac{K_1}{s^3 + 3s^2 + (2+K_1 K_2)s + K_1}$$

The characteristic equation of the system is

$$s^3 + 3s^2 + (2+K_1 K_2)s + K_1 = 0$$

$$\text{Let } \alpha = K_1 K_2$$

$$\beta = K_1$$

The equation becomes:

$$s^3 + 3s^2 + (2+\alpha)s + \beta = 0$$

From the parameter plane method

$$B_1 = -\zeta\omega_n$$

$$B_2 = -\omega_n$$

$$C_1 = 1$$

$$C_2 = 0$$

$$D_1 = (4\zeta^3 - 3\zeta)(-\omega_n^3) + (2\zeta^2 - 1)(3\omega_n^2) + (-2\zeta\omega_n)$$

$$D_2 = (4\zeta^2 - 1)(-\omega_n^3) + (2\zeta)(3\omega_n^2) + (-2\omega_n)$$

So, the solutions for α and β are

$$\alpha = -4\omega_n^2\zeta^2 + 6\omega_n\zeta + \omega_n^2 - 2$$

$$\beta = 3\omega_n^2 - 2\zeta\omega_n$$

The ζ -curves and ω_n -curves on the α - β plane are shown on Figure 2.1.

Illustration II. The system is the same as illustration I

let $K_1 = \beta$

$K_2 = \alpha$

The characteristic equation becomes:

$$s^3 + 3s^2 + (2+\alpha\beta)s + \beta = 0$$

Then

$$B_1 = 0 \quad B_2 = 0$$

$$C_1 = 1 \quad C_2 = 0$$

$$H_1 = -\zeta\omega_n \quad H_2 = -\omega_n$$

$$D_1 = (3\zeta - 4\zeta^3)\omega_n^3 + (6\zeta^2 - 3)\omega_n^2 - 2\zeta\omega_n$$

$$D_2 = (1 - 4\zeta^2)\omega_n^3 + 6\zeta\omega_n^2 - 2\omega_n$$

And the other definitions are

$$a = 0; \quad b = \omega_n; \quad c = 0;$$

$$d = -4\omega_n^3\zeta^3 + 6\omega_n^2\zeta^2 + (3\omega_n^3 - 2\omega_n)\zeta - 3\omega_n^2$$

$$e = f = 2\omega_n^4\zeta - 3\omega_n^3$$

So only one solution for α and β in equations 2.6a and 2.6b is obtained:

$$\beta = 3\omega_n^2 - 2\omega_n^3 \zeta$$

$$\alpha = \frac{4\omega_n^2 \zeta^2 - 6\omega_n \zeta - \omega_n^2 + 2}{2\omega_n^3 \zeta - 3\omega_n^2} = \frac{2\zeta}{\omega_n} + \frac{\omega_n^2 - 2}{\beta}$$

The curves are shown on Figure 2.2

Consider the curves on Figures 2.1 and 2.2: the roots of the characteristic equation are defined as functions of α and β . If we choose $\beta = 40$ and $\alpha = 20$ on Figure 2.1, then

$$\omega_n = 4.5$$

$$\zeta = 0.111$$

therefore, the roots of the system are:

$$S = -0.5 \pm j4.47$$

and the corresponding value for the real root is -2. In like manner on Figure 2.2, if we desire roots at $\zeta = 0.111$ and $\omega_n = 4.5$ we found $\alpha = 0.5$ and $\beta = 40$.

On the other hand, if $\zeta = 0.2$ and $\omega_n = 5$ the roots are $-1 \pm j4.9$ and -1. From the Figure we find:

$$\beta = K_1 = 25$$

$$\alpha = 25 \text{ (Figure 2.1) or } 1.0 \text{ (Figure 2.2) therefore } K_2 = 1.0$$

Parameter plane methods are very convenient. The α - β plane can be used to find the system parameters, and/or roots of the characteristic equation at any defined values of the parameter. The stability of the system can be investigated at the line $\zeta=0$ which corresponds to the imaginary axis on the S-plane.

$$\begin{aligned}
 T_0 &= 1 & U_0 &= 0 \\
 T_1 &= \zeta & U_1 &= 1 \\
 T_2 &= 2\zeta^2 - 1 & U_2 &= 2\zeta \\
 T_3 &= 4\zeta^3 - 3\zeta & U_3 &= 4\zeta^2 - 1 \\
 T_4 &= 8\zeta^4 - 8\zeta^2 + 1 & U_4 &= 8\zeta^3 - 4\zeta \\
 T_5 &= 16\zeta^5 - 20\zeta^3 + 5\zeta & U_5 &= 16\zeta^4 - 12\zeta^2 + 1 \\
 T_6 &= 32\zeta^6 - 48\zeta^4 + 18\zeta^2 - 1 & U_6 &= 32\zeta^5 - 32\zeta^3 + 6\zeta \\
 T_7 &= 64\zeta^7 - 112\zeta^5 + 56\zeta^3 - 7 & U_7 &= 64\zeta^6 - 80\zeta^4 + 24\zeta^2 - 1 \\
 T_8 &= 128\zeta^8 - 256\zeta^6 + 160\zeta^4 - 32\zeta^2 + 7 & U_8 &= 128\zeta^7 - 192\zeta^5 + 80\zeta^3 - 8\zeta \\
 T_9 &= 256\zeta^9 - 576\zeta^7 + 432\zeta^5 - 120\zeta^3 + 9 & U_9 &= 256\zeta^8 - 448\zeta^6 + 240\zeta^4 - 40\zeta^2 + 1 \\
 T_{10} &= 512\zeta^{10} - 1280\zeta^8 + 1120\zeta^6 - 400\zeta^4 + 50\zeta^2 - 1 & U_{10} &= 512\zeta^9 - 1024\zeta^7 + 672\zeta^5 - 160\zeta^3 + 10
 \end{aligned}$$

Table 2. Functions $T_k(\xi)$

ξ	T_0	T_1	T_2	T_3	T_4	T_5	T_6
0.00	0.00	-1.000	0.0000	1.00000	0.000000	-1.0000000	
0.05	0.05	-0.995	-0.1495	0.98005	0.347505	-0.9452995	
0.10	0.10	-0.980	-0.2960	0.92080	0.480160	-0.8247680	
0.15	0.15	-0.955	-0.4365	0.82405	0.683715	-0.6189355	
0.20	0.20	-0.920	-0.5680	0.69280	0.845120	-0.3547520	
0.25	0.25	-0.875	-0.6875	0.53125	0.953125	-0.0546875	
0.30	0.30	-0.820	-0.7920	0.34480	0.998880	0.2545280	
0.35	0.35	-0.755	-0.8785	0.14005	0.976535	0.5435245	
0.40	0.40	-0.680	-0.9440	-0.07520	0.883840	0.7822720	
0.45	0.45	-0.595	-0.9855	-0.29195	0.722745	0.9424205	
0.50	0.50	-0.500	-1.0000	-0.50000	0.500000	1.0000000	
0.55	0.55	-0.395	-0.9845	-0.68795	0.227755	0.9384805	
0.60	0.60	-0.280	-0.9360	-0.84320	-0.075840	0.7521920	
0.65	0.65	-0.155	-0.8515	-0.95195	-0.386035	0.4501045	
0.70	0.70	-0.020	-0.7280	-0.99920	-0.670880	0.0599680	
0.75	0.75	0.125	-0.5625	-0.96875	-0.890625	-0.3671875	
0.80	0.80	0.280	-0.3520	-0.84320	-0.997120	-0.7521920	
0.85	0.85	0.445	-0.0935	-0.60395	-0.933215	-0.9825155	
0.90	0.90	0.620	0.2160	-0.23120	-0.632160	-0.9066880	
0.95	0.95	0.805	0.5795	0.29605	-0.017005	-0.3283595	
1.00	1.00	1.000	1.0000	1.00000	1.000000	1.0000000	

T_5	T_4	T_5	T_{10}
0.000000000	1.000000000	0.000000000	-1.000000000
-0.44203495	0.901096005	0.5321445505	-1.84788154995
-0.64511360	0.695745280	0.7842626560	-0.53889274880
-0.86939565	0.358116805	0.9768306915	-0.06506759755
-0.98702080	-0.040056320	0.9709982720	0.42845562880
-0.98046875	-0.435546875	0.7626953125	0.81689453125
-0.84616320	-0.762225920	0.3888276480	0.99552250880
-0.59606785	-0.960771995	-0.0764725465	0.90724121245
-0.25802240	-0.988689920	-0.5329295360	0.46234629120
0.12543345	-0.829530395	-0.8720108055	0.04472067005
0.50000000	-0.500000000	-1.000000000	-0.500000000
0.80457355	-0.053449595	-0.8633668104	-0.98625531995
0.97847040	0.421972480	-0.4721034240	-0.98849658880
0.97117085	0.812407605	0.0849590365	-0.74178047245
0.75483520	0.996801280	0.6406865920	-0.09984005120
0.33984375	0.876953125	0.9755859375	0.58642578125
-0.20638720	0.421972480	0.8815431680	0.98849658880
-0.73706135	-0.270488795	0.2772303985	0.74178047245
-0.99987840	-0.893093120	-0.6076892160	-0.20074746880
-0.60687805	-0.824708795	-0.9600686605	-0.99942165995
1.00000000	1.000000000	1.000000000	1.000000000

Table 3. Functions $U_k(\xi)$

ξ	U_{-1}	U_0	U_1	U_2	U_3	U_4	U_5	U_6
0.00				0.0	-1.00	0.000	1.0000	0.00000
0.05				0.1	-0.99	-0.199	0.9701	0.29601
0.10				0.2	-0.96	-0.392	0.8816	0.56832
0.15				0.3	-0.91	-0.573	0.7381	0.79443
0.20				0.4	-0.84	-0.736	0.5456	0.95424
0.25				0.5	-0.75	-0.875	0.3125	1.03125
0.30				0.6	-0.64	-0.984	-0.0496	1.01376
0.35				0.7	-0.51	-0.057	-0.2299	0.89607
0.40				0.8	-0.36	-0.088	-0.5104	0.67968
0.45				0.9	-0.19	-0.071	-1.7739	0.37449
0.50	-1	0	1	1.0	0.00	-1.000	-1.0000	0.00000
0.55				1.1	0.21	-0.869	-1.1659	-0.41349
0.60				1.2	0.44	-0.672	-1.2464	-0.82368
0.65				1.3	0.69	-0.403	-1.2139	-0.75071
0.70				1.4	0.96	-0.056	-1.0384	-1.39776
0.75				1.5	1.25	0.375	-0.6875	-1.40625
0.80				1.6	1.56	0.896	-0.1264	-1.09824
0.85				1.7	1.89	1.513	0.6821	-0.35343
0.90				1.8	2.24	2.232	1.7776	0.69768
0.95				1.9	2.61	3.059	3.2021	3.02499
1.00				2.0	3.00	4.000	5.0000	6.00000
	U_7	U_8		U_9		U_{10}		
-1.000000	0.0000000		1.00000000		0.000000000			
-0.940499	-0.3900599		0.90149301		0.480209201			
-0.767936	-0.3900599		0.62355456		0.846618112			
-0.499771	-0.7219072		0.21264621		1.009300083			
-0.163904	-1.9443613		-0.24401664		0.922194944			
0.203125	-0.0198016		-0.66706875		0.595703125			
0.558656	-0.9296875		-0.96579584		0.099088896			
0.857149	-0.6785664		-1.06439499		-0.449010793			
1.054144	0.2060657		-0.92323584		-0.902223872			
1.110941	0.6253569		-0.54811979		-1.118664711			
1.000000	1.0000000		0.00000000		-1.000000000			
0.711061	1.1956571		0.60416981		-0.531079109			
0.257984	1.1132608		1.10192896		0.189053952			
-0.313691	0.7672717		1.31114421		0.937215773			
-0.981464	0.1119104		1.07513856		1.393283584			
-1.421876	-0.7265625		0.33203125		1.224609375			
-1.630784	-1.5110144		-0.78683904		0.252071936			
-0.292931	-1.8275527		-1.82390859		-1.273091903			
-0.035776	-1.0320768		-1.82196224		-2.247455232			
2.545381	1.9112339		0.99596341		-0.108903421			
7.000000	8.0000000		9.00000000		10.000000000			

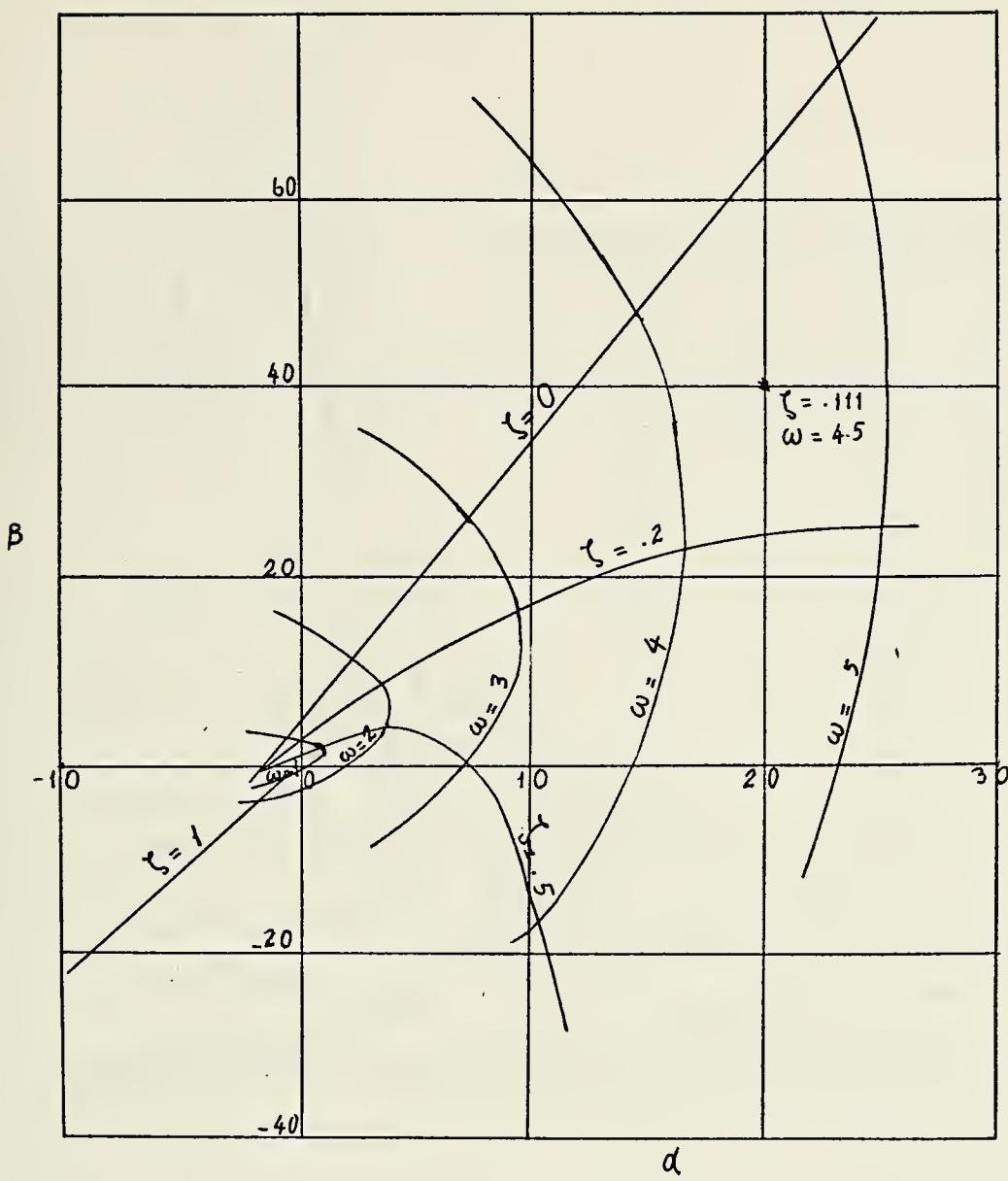


Figure 2.1 Root Parameter Plane Curves for illustration 1.

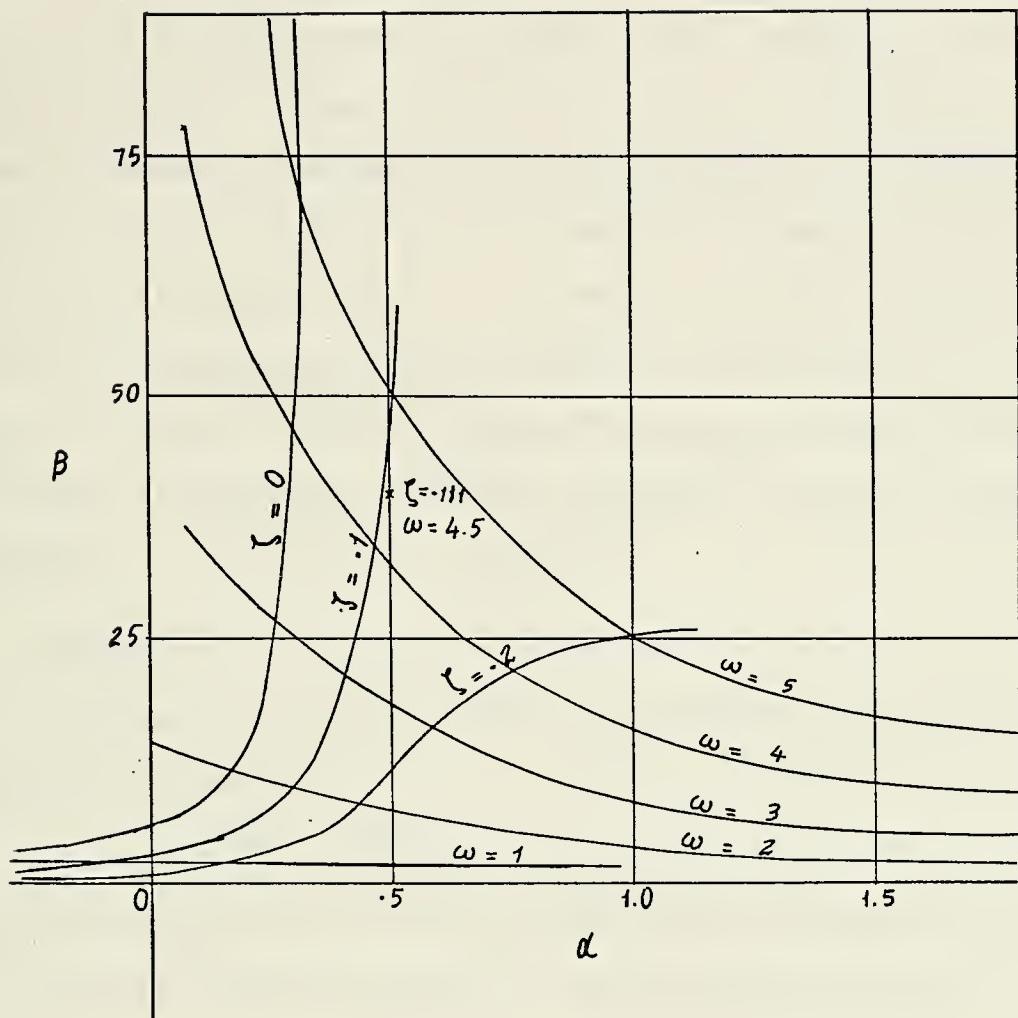


Figure 2.2 Root Parameter Plane Curves for Illustration 2.

III. PARAMETER FREQUENCY RESPONSE TECHNIQUES

A. INTRODUCTION

Both linear and non linear dynamic systems may be subjected to periodic inputs. It is important therefore that we be able to analyze such systems in the frequency domain. If the system is linear there also exists a correlation between frequency response and time response as guaranteed by the Fourier integral which permits us to estimate the transient response from the frequency response. Thus the frequency domain may be used to design systems for non-periodic inputs. In this chapter frequency response analysis of linear systems on a frequency parameter plane is discussed in detail.

B. ANALYSIS

A linear system can be described mathematically by a transmission function:

$$T(s) = \frac{N(s)}{D(s)}$$

where the system may be either open-loop or closed-loop and $N(s)$ and $D(s)$ are polynomials. In polynomial notation

$$T(s) = \frac{\sum_{k=0}^m a_k s^k}{\sum_{q=0}^n b_q s^q}$$

where, a 's and b 's are coefficients which are functions of the system parameters.

If we choose two systems parameters α and β and if we consider only sinusoidal signals the frequency response transmission function becomes

$$\begin{aligned} T(j\omega) &= \frac{N(j\omega, \alpha, \beta)}{D(j\omega, \alpha, \beta)} \\ &= \frac{a_0 + a_1 j\omega + a_2 (j\omega)^2 + \dots + a_m (j\omega)^m}{b_0 + b_1 j\omega + b_2 (j\omega)^2 + \dots + b_n (j\omega)^n} \\ &= \frac{[a_0 - a_2 \omega^2 + a_4 \omega^4 - \dots] + j[a_1 \omega - a_3 \omega^3 + a_5 \omega^5 - \dots]}{[b_0 - b_2 \omega^2 + b_4 \omega^4 - \dots] + j[b_1 \omega - b_3 \omega^3 + b_5 \omega^5 - \dots]} \end{aligned}$$

In abbreviated notation

$$T(j\omega) = \frac{N_e(\omega, \alpha, \beta) + jN_o(\omega, \alpha, \beta)}{D_e(\omega, \alpha, \beta) + jD_o(\omega, \alpha, \beta)}$$

Similarly

$$T(-j\omega) = \frac{N_e(\omega, \alpha, \beta) - jN_o(\omega, \alpha, \beta)}{D_e(\omega, \alpha, \beta) - jD_o(\omega, \alpha, \beta)}$$

Let the magnitude be represented by M :

$$M = |T(j\omega)|$$

$$M^2 = |T(j\omega)|^2 = |T(j\omega)| |T(-j\omega)|$$

therefore

$$M^2 = \frac{[N_e(\omega, \alpha, \beta)]^2 - [N_o(\omega, \alpha, \beta)]^2}{[D_e(\omega, \alpha, \beta)]^2 - [D_o(\omega, \alpha, \beta)]^2}$$

from which

$$M^2 = \frac{[a_0 - a_2 \omega^2 + a_4 \omega^4 - \dots]^2 + [a_1 \omega - a_3 \omega^3 + \dots]^2}{[b_0 - b_2 \omega^2 + b_4 \omega^4 - \dots]^2 + [b_1 \omega - b_3 \omega^3 + \dots]^2}$$

$$\begin{aligned} &= \frac{\sum_{k=0}^{2m} (-1)^{\frac{k}{2}} a_k \omega^k + \sum_{k=1}^{2m-1} (-1)^{\frac{k-1}{2}} a_k \omega^k}{\sum_{q=0}^{2n} (-1)^{\frac{q}{2}} b_q \omega^q + \sum_{q=1}^{2n-1} (-1)^{\frac{q-1}{2}} b_q \omega^q} \end{aligned}$$

Then

$$M^2 = \frac{\sum_{k=0}^m (-1)^k a_k'' \omega^{2k}}{\sum_{q=0}^n (-1)^q b_q'' \omega^{2q}}$$

where, the a_k'' 's and b_q'' 's are functions of α and β . So, there are four variables M , ω , α , and β in this equation. The equation can be plotted by fixing one variable and using one variable as a parameter. Seven different combinations are possible.

Example. Consider the system of a Steam engine Governor [9] for which the transmission function can be written as:

$$\frac{\phi}{\phi_c} = \frac{\frac{C_1 C_2}{I m} s}{s^4 + (\frac{C_g + C_e}{m} I) s^3 + (\frac{K_e + K_g}{I m} + \frac{C_e C_g}{I m}) s^2 + (\frac{K_e C_g}{I m} + \frac{C_e K_g}{I m} + \frac{C_1 C_2}{I m}) s + \frac{K_e K_g}{I m}}$$

where ϕ = instantaneous speed response.

ϕ_c = command speed.

m = equivalent mass of the governor sleeve.

C_g = damping coefficient at the governor sleeve.

K_g = stiffness of governor spring.

C_1 = increase in the upward force on the governor sleeve.

C_2 = increase in steam torque.

I = equivalent moment of inertia of the engine.

C_e = damping coefficient of the engine.

K_e = stiffness of engine spring.

From studying the mechanical characteristics [10], possible values of the parameters of the system are substituted in the equation. Let K_g and C_2 be varied by adjusting the governor spring and the valves for steam flow respectively. Then the equation becomes:

$$\frac{\phi}{\phi_c} = \frac{626\alpha s}{s^4 + 10.5s^3 + (30+25\beta)s^2 + (137.5\beta + 626\alpha + 12.5)s + 62.5\beta}$$

where $\alpha = C_2$

$\beta = K_g$

then

$$M^2 = \frac{[626\alpha\omega]^2}{[\omega^4 - (30+25\beta)\omega^2 + 62.5\beta]^2 + [-10.5\omega^3 - (137.5\beta + 626\alpha + 12.5)\omega]^2}$$

From the curves of Figure 3.1 several useful bits of information can be obtained by inspection:

(a) At any intersection between two constant ω curves the coordinates define the α - β pair, and the values of ω of the intersecting curves define the bandwidth (since the magnitude is -3 db).

(b) A locus of constant ω define a set of α , β pairs which are potentially useful in self adaptive applications, i.e., if, in a given system, β is subject to change, values of α can be determined such that a magnitude $M = M_1$ can be maintained at $\omega = \omega_1$, by determining the values assumed by β and providing a suitable adjustment in α .

(c) If a number of families of curves are generated with coordinates as in Figure 3.1 but with different M values, the entire frequency response can be determined for any chosen α , β pair.

From the curves of Figure 3.2a, b and c, it can be seen that:

(a) Each frequency has only one α , β pair which gives the maximum magnitude.

i.e., at $\omega = 10$, $\alpha = 1.0$, $\beta = 3.0$ (Figure 3.2a)

$\omega = 12$, $\alpha = 1.4$, $\beta = 4.5$ (Figure 3.2b)

$\omega = 14$, $\alpha = 1.75$, $\beta = 6.6$ (Figure 3.2c)

(b) The locus of the maximum magnitude point is a straight line when frequency is increased.

(c) The axis which is drawn through the ellipses gives the values of β (and/or α) and magnitude which are coordinates of points on the envelope of the α versus magnitude curve as in Figure 3.3.

On Figure 3.3 are curves of α versus magnitude. These curves as drawn show the envelope of the family of curves that corresponds to the axis of the ellipses on Figure 3.2. This envelope bounds all possible magnitudes at a frequency of 12 rad/sec, for all β .

Figure 3.4, which presents a family of constant magnitude curves on α versus frequency coordinates ($\beta = \text{constant}$) provides a global view of the possible frequency responses of the system:

(a) We can plot the Bode diagram by reading the frequency and magnitude at the intersections along any line of constant α .

(b) The maximum magnitude for the given values of β is found at the center of the family of ellipses, and also increases as a straight line when β is increased.

(c) At constant β ($\beta=5.0$), the curve of magnitude is -3 db., it shows the bandwidth is between the intersection of the curve and any line of constant α , such as at $\alpha = 2$ the bandwidth is 10.5 rad/sec.

A plot of β versus frequency with constant α may be used in the same way.

Curves of α versus magnitude are shown on Figure 3.5 with β constant and frequency the family parameter.

(a) The Bode diagram can be plotted at any α by evaluating magnitude and frequency along a horizontal line.

(b) At the intersections of two frequency curves the Bode diagram for that value of α will have the same magnitude at each of the frequencies.

(c) Each frequency curve defines one value of α (β is constant) for which the magnitude is maximum at that frequency.

(d) If two frequency curves intersect each other at the magnitude -3 db., the bandwidth at that value of α is defined by the two frequencies.

Figure 3.6 plots α versus frequency with β as the parameter. Each β -curve is intersected twice by each α line, thus defining two values of frequency for the one value of the α , β pair, these two values of frequency give the bandwidth of the system (since the magnitude is -3 db).

Figure 3.7 shows a family of Bode diagrams with α as the family parameter and β constant. It may be used to select a value of α , or to define a desired variation in α as a function of frequency.

Each curve family is seen to contain useful information. Depending on the type of information needed some curve families may be more useful than others.

C. STABILITY ON THE FREQUENCY PARAMETER PLANE

Figure 3.8 shows the stability on the root parameter plane of the above system. The curve of $\zeta = 0$ is obtained from:

$$\alpha = \frac{1.248 - 0.4902\omega^2 + 5.541\omega^3 + 0.19968\omega^4}{\omega(25\omega^2 - 62.5)}$$

$$\beta = \frac{\omega^4 - 30\omega^2}{25\omega^2 - 62.5}$$

In the typical system, the region of stability on the frequency parameter plane can be determined by using the above relations.

The stability boundary ($\zeta=0$) curve has been added on Figure 3.1, through Figure 3.6 and the shaded areas are the stable region. Synthesis and design with the frequency parameter plane are considered only for stable systems.

Each Figure has a different interpretation. Figure 3.1 plots α versus β with magnitude constant at -3 db, and frequency as the family parameter. By choosing an operating point in the stable region at the intersection of two frequency curves, one guarantees both stability and bandwidth. Figure 3.2 uses frequency as the constant parameter and magnitude as the family parameter. This permits optimum choice of α and β for desire magnitude and frequency.

The other curves can be considered in the same way. At any point on the Figures in the stable region, the parameters at that point can be used to design the system and obtain values of magnitude and frequency as desired.

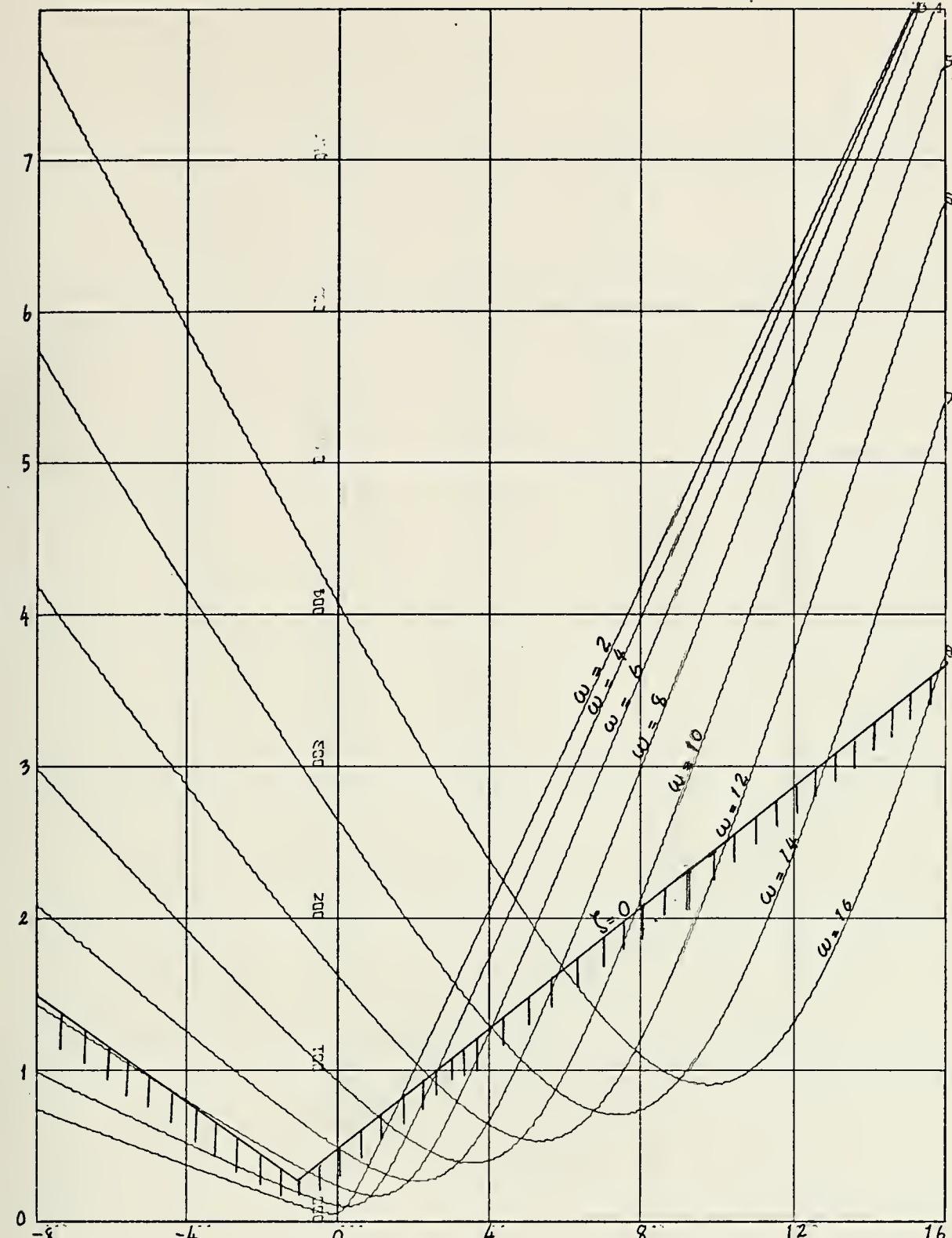


Figure 3.1 Frequency Parameter Plane with Stability Limits.
Plot of α vs β with M constant at -3 db and ω
as family parameter.

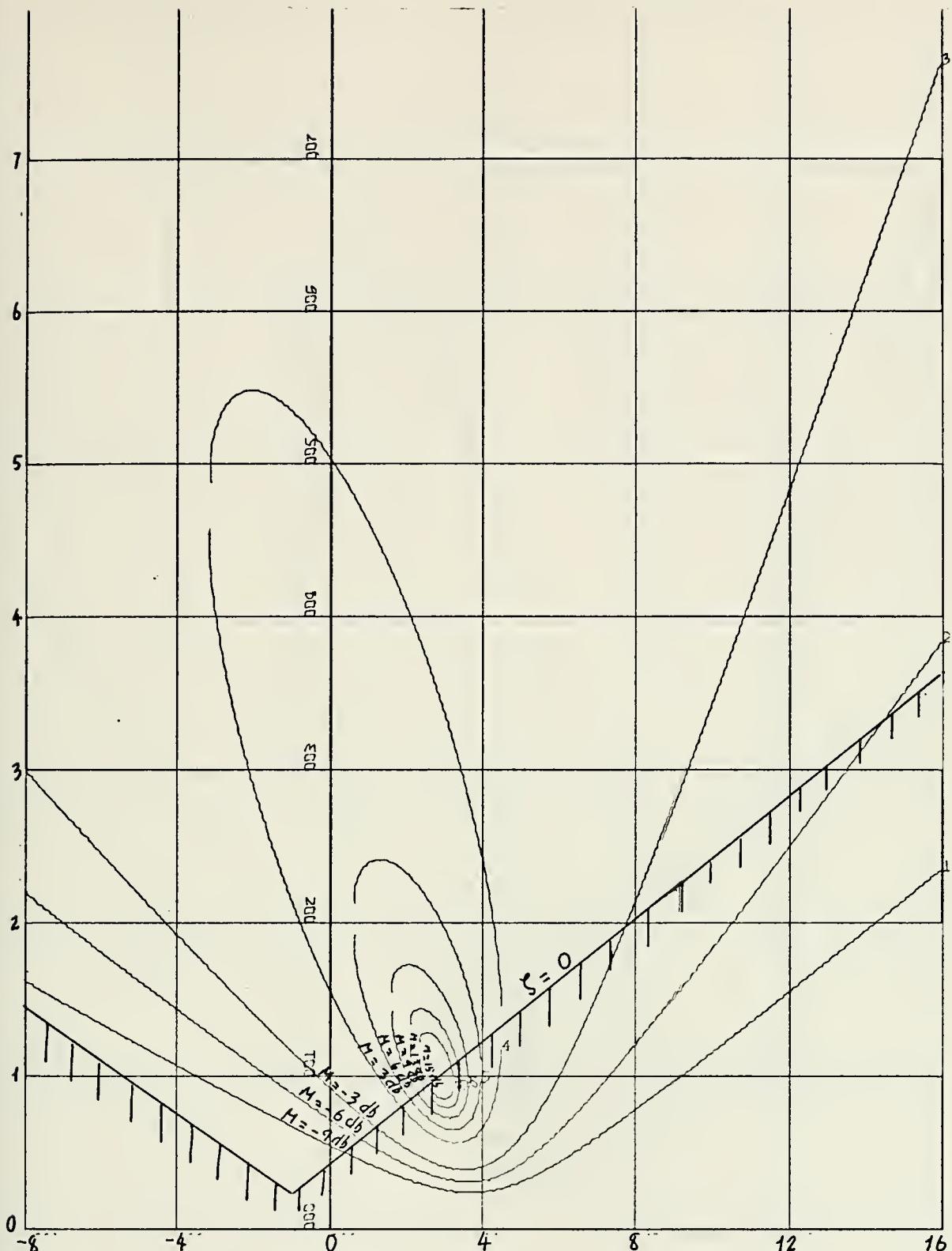


Figure 3.2a Frequency Parameter Plane with Stability Limits.
Plot of α vs β with ω constant at 10 rad/sec
and M as family parameter.

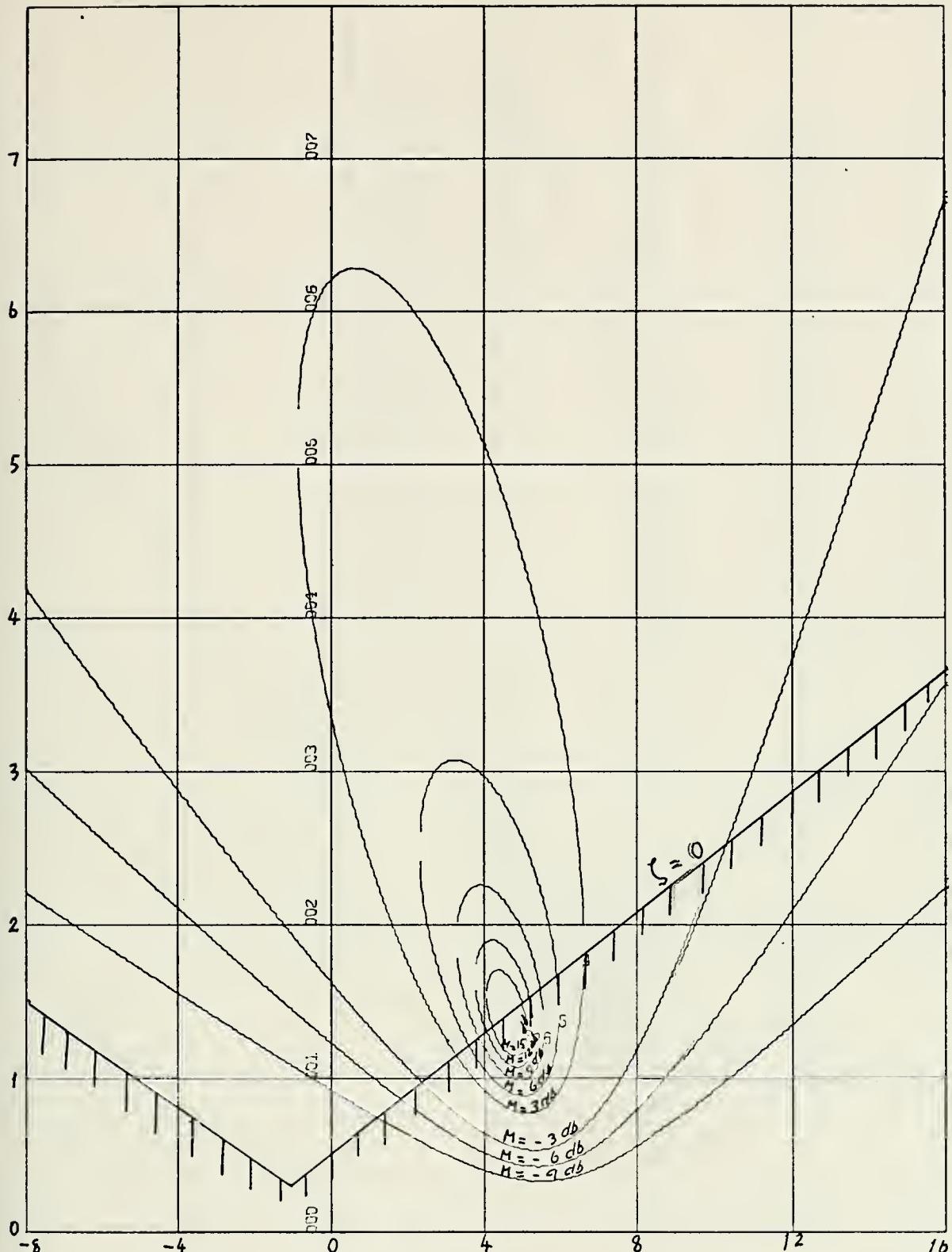


Figure 3.2b Frequency Parameter Plane with Stability Limits.
Plot of α vs β with ω constant at 12 rad/sec
and M as family parameter.

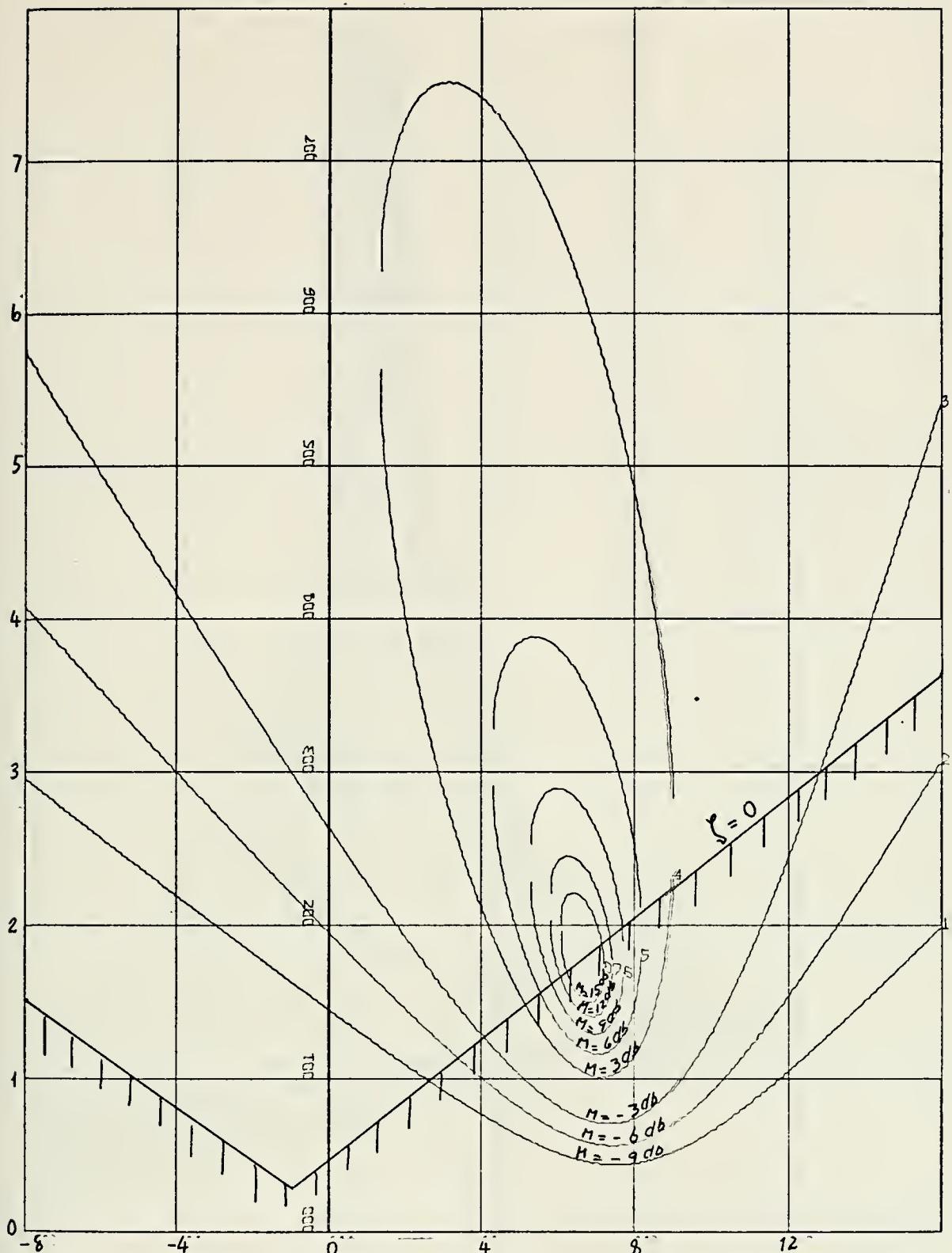
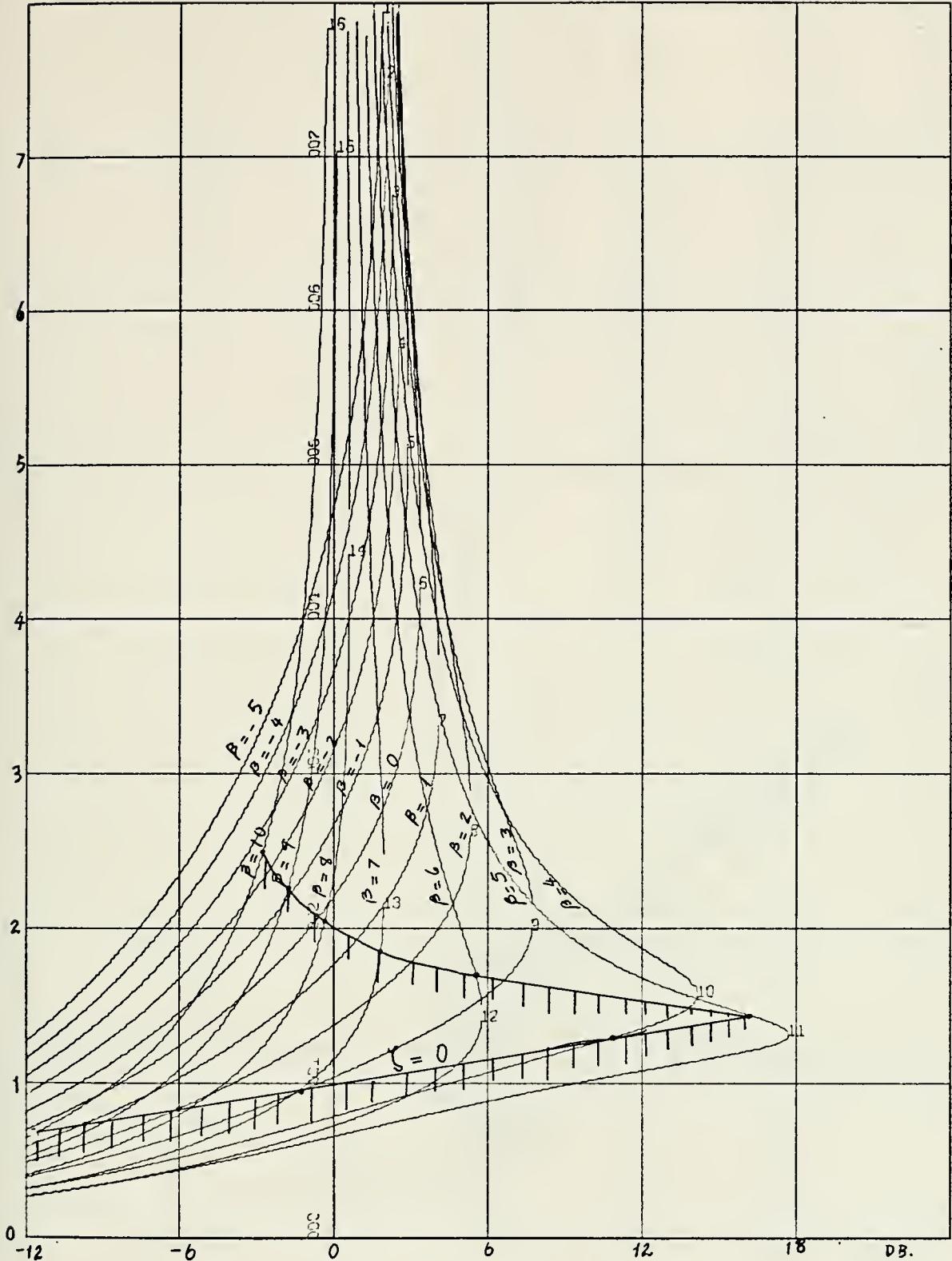


Figure 3.2c Frequency Parameter Plane with Stability Limits.
Plot of α vs β with ω constant at 14 rad/sec
and M as family parameter.



**Figure 3.3 Frequency Parameter Plane with Stability Limits.
Plot of α vs M with ω constant at 12 rad/sec
and β as family parameter.**

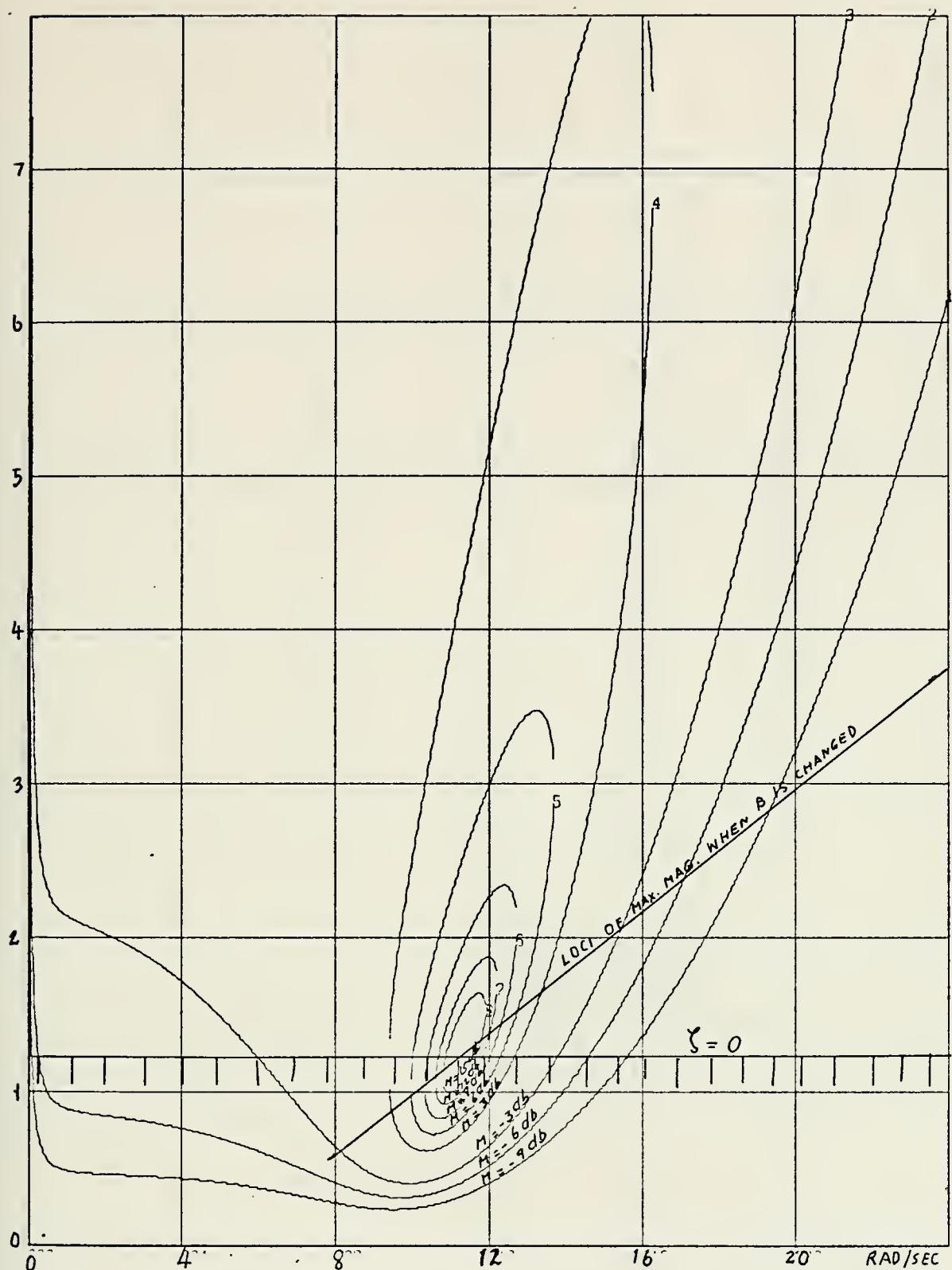


Figure 3.4a Frequency Parameter Plane with Stability Limits.
Plot of α vs ω with β constant at $\beta=4$ and M as family parameter.

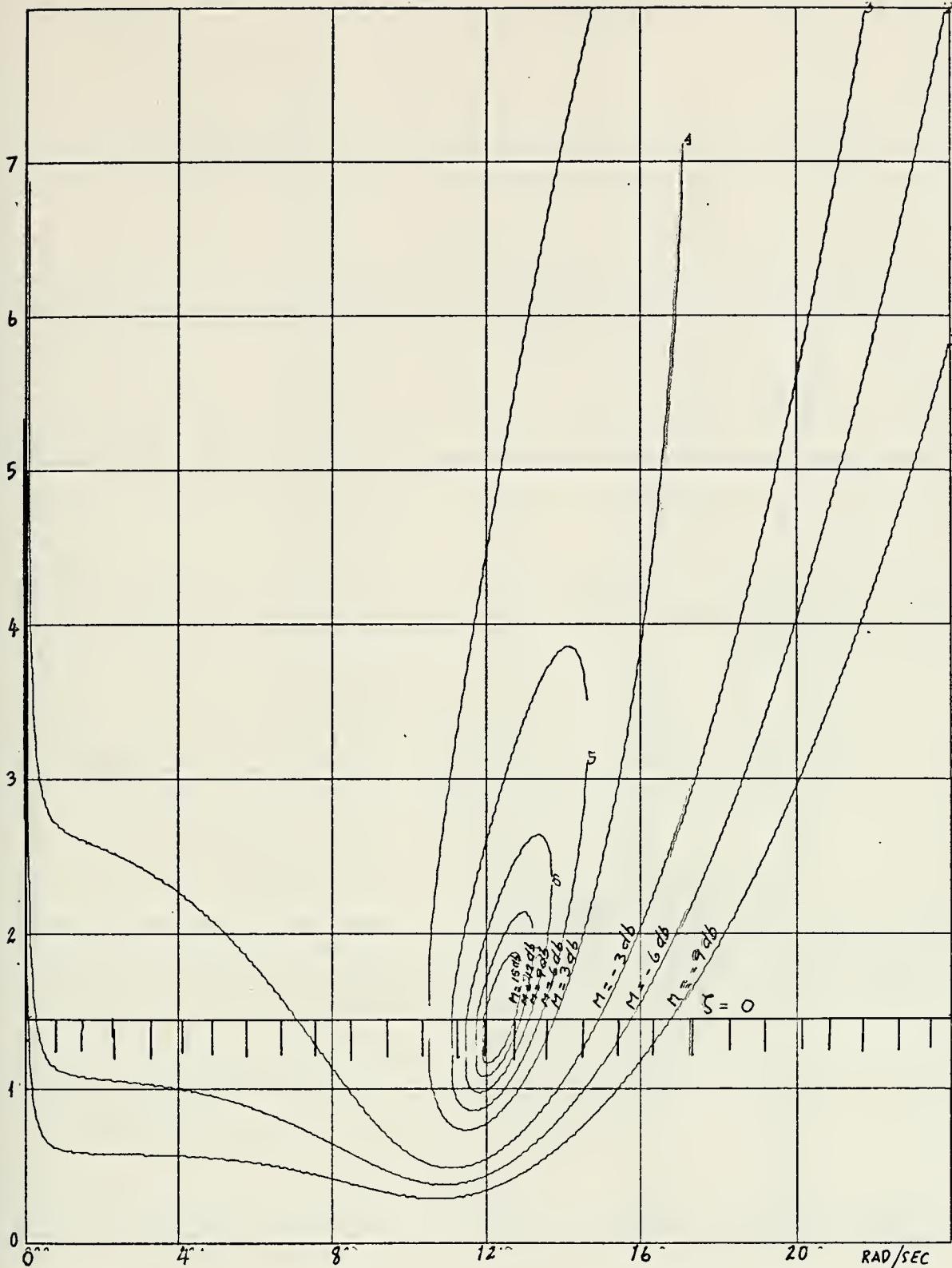


Figure 3.4b Frequency Parameter Plane with Stability Limits.
Plot of α vs ω with $\beta=5$ and M as family parameter

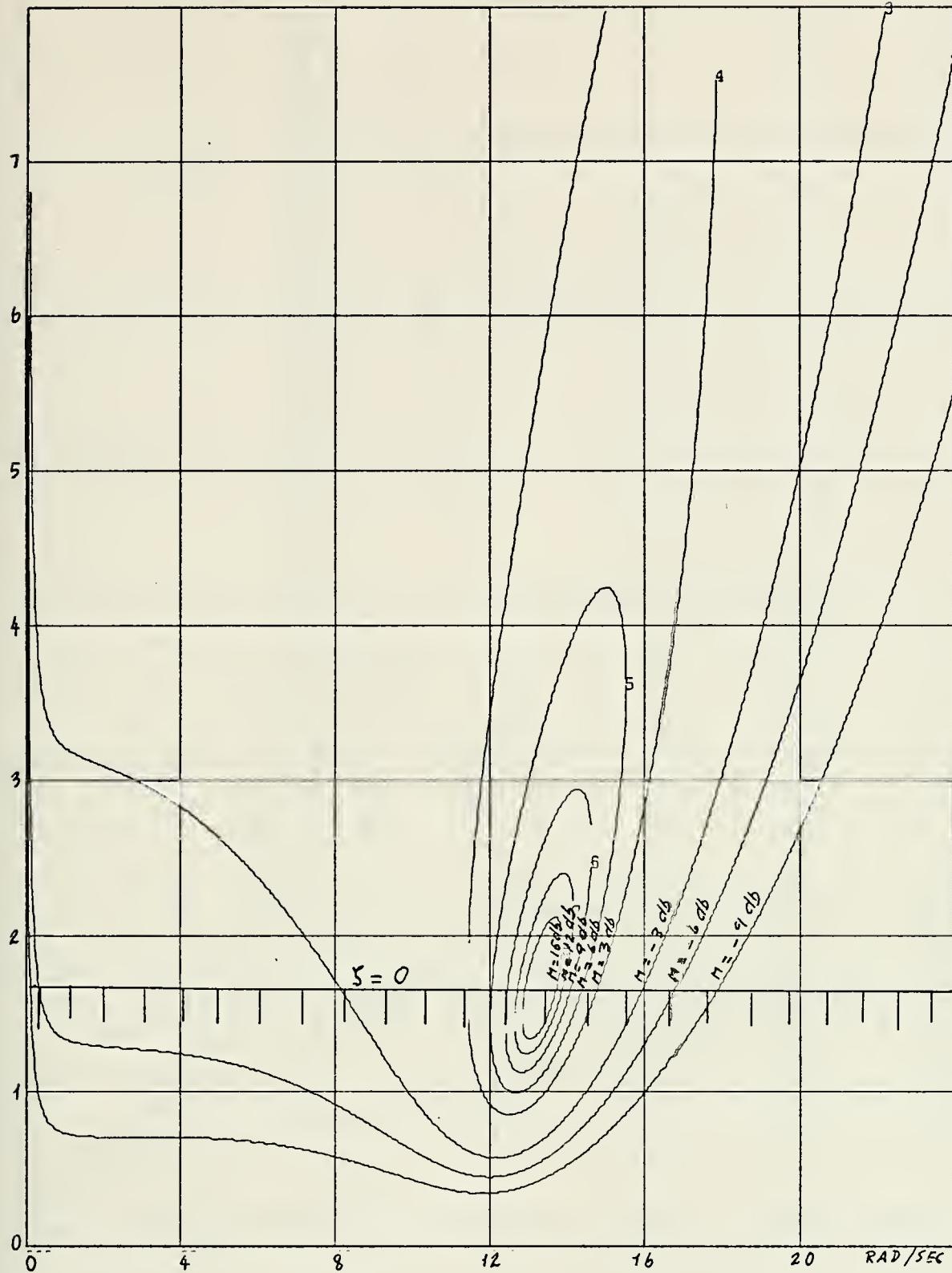


Figure 3.4c Frequency Parameter Plane with Stability Limits.
 Plot of α vs ω with β constant at $\beta=6$ and M as family parameter.

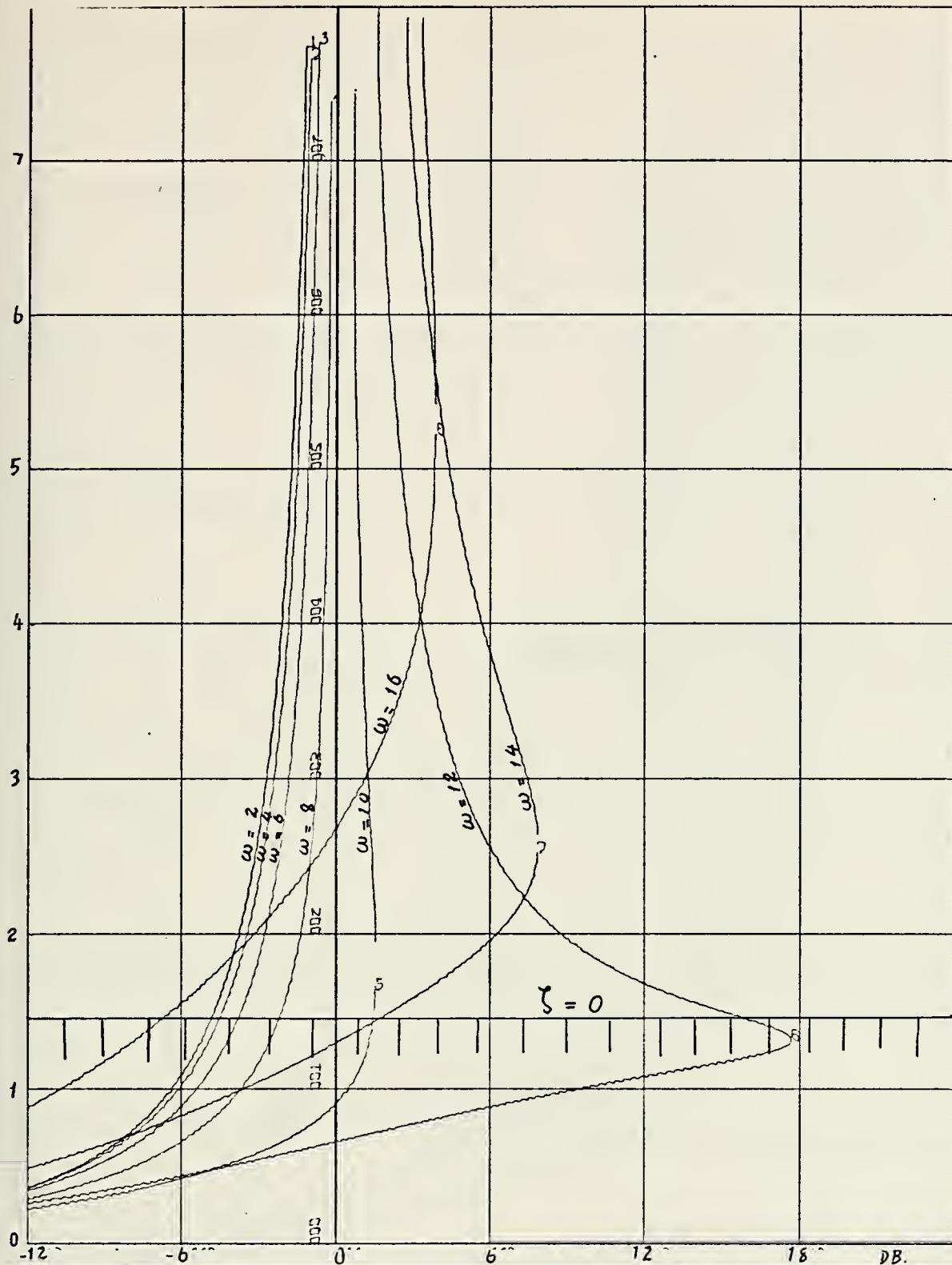


Figure 3.5 Frequency Parameter Plane with Stability Limits.
Plot of α vs M with β constant at $\beta=5$ at ω as family parameter.

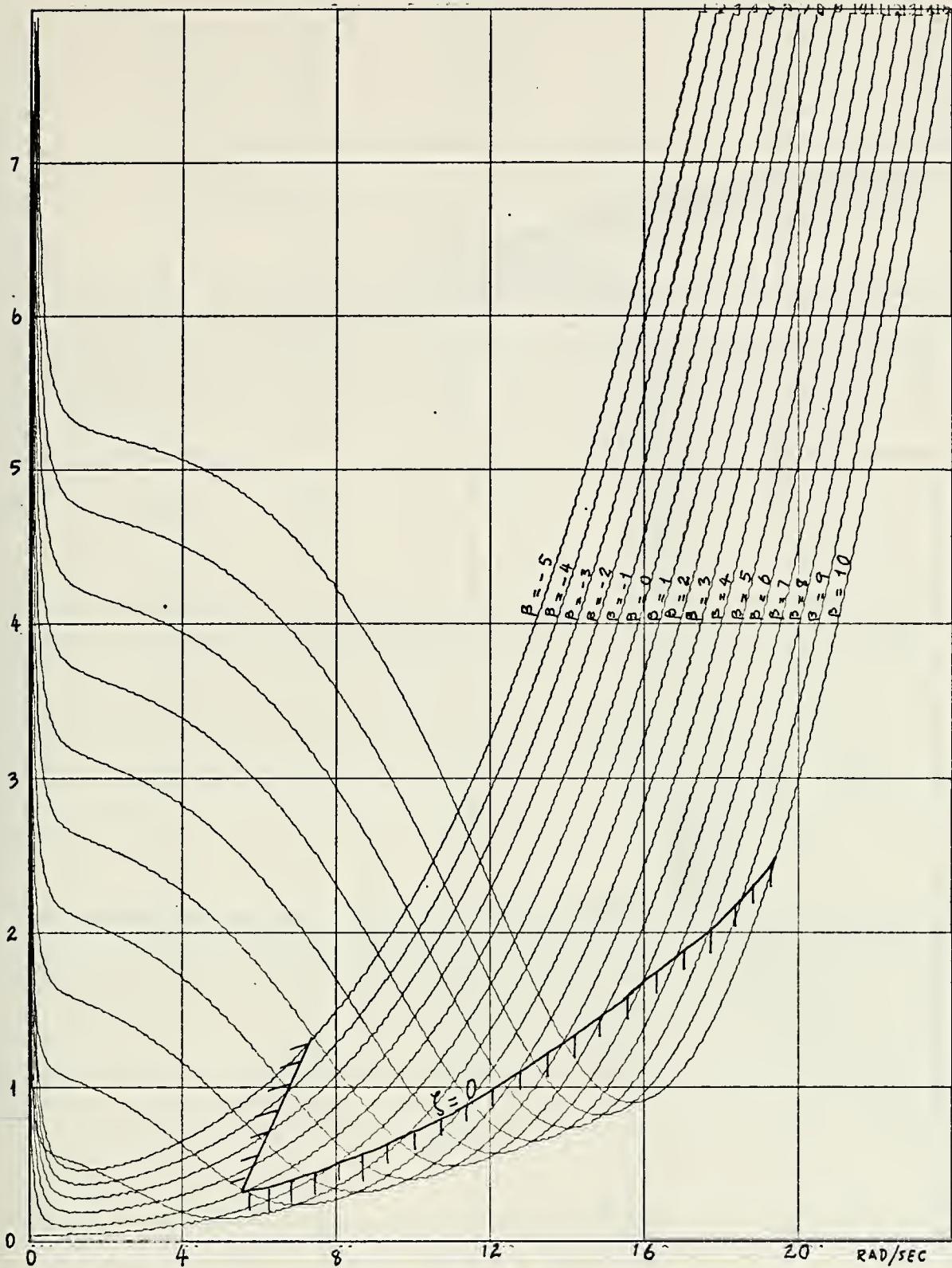


Figure 3.6 Frequency Parameter Plane with Stability Limits.
Plot of α vs ω with M constant at -3 db and β as the family parameter.

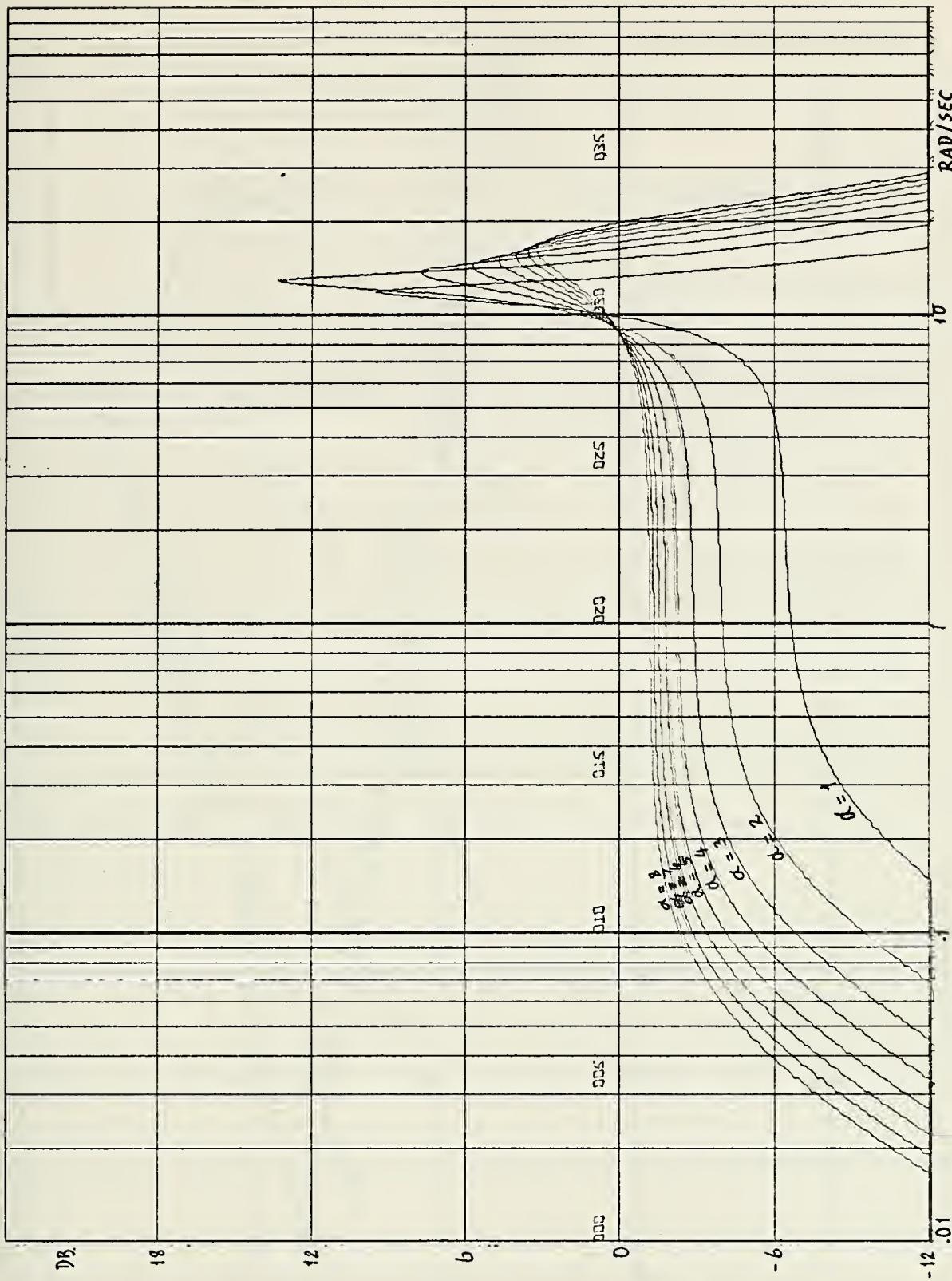


Figure 3.7 Frequency Parameter Plane (Bode Diagram). Plot of M vs ω with β constant at $\beta=5$ and α as family parameter.

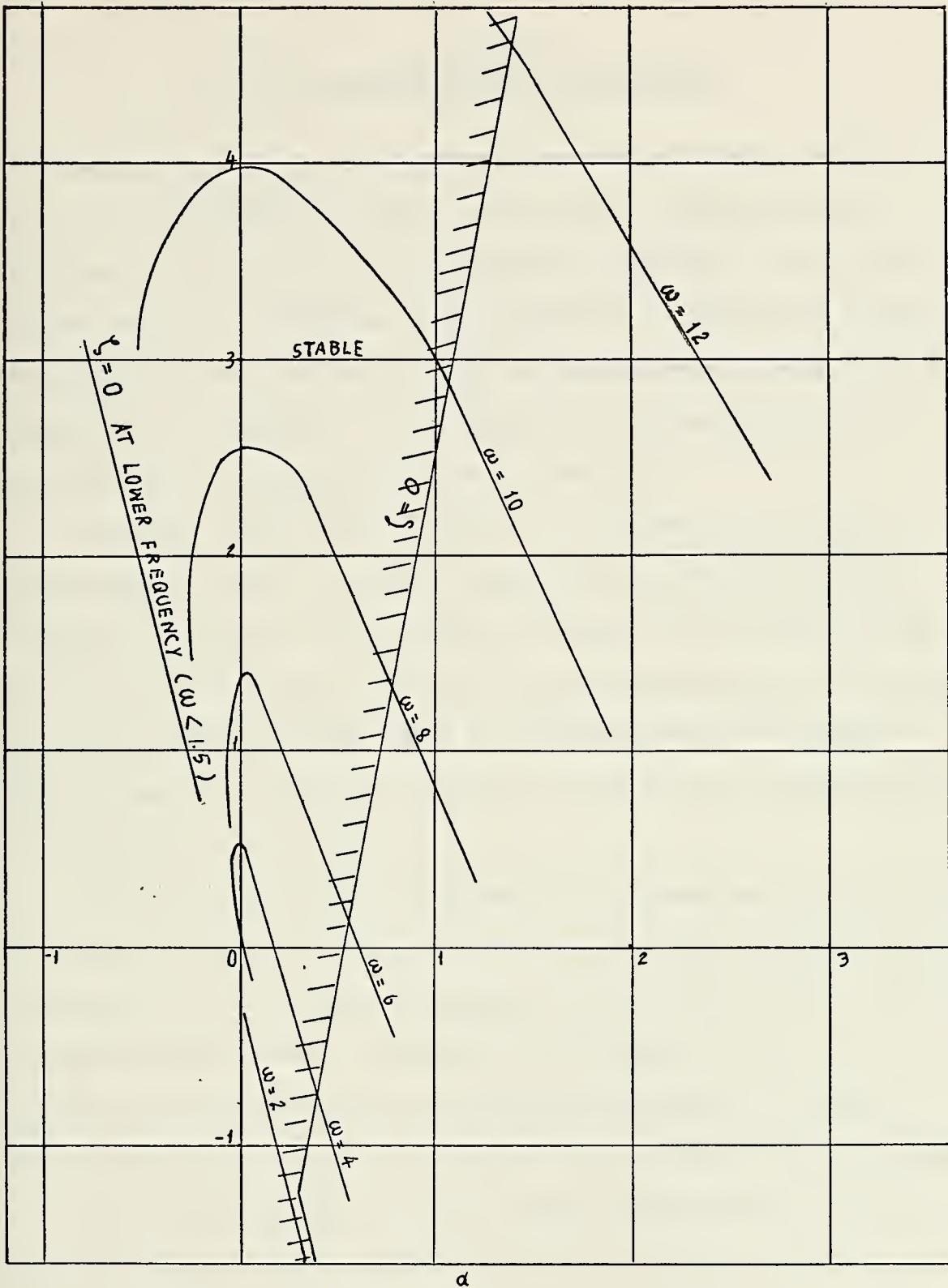


Figure 3.8 Root Parameter Plane Plot.

IV. NONLINEAR SYSTEM TECHNIQUES

Generally, nonlinear systems consist of linear parts which are described by linear differential equations and nonlinear parts which are described by nonlinear functions. If the system is nonlinear with an applied sinusoidal input, the signal at various points in the system may or may not be sinusoidal. In general the signals will be periodic, but may contain sub-harmonics and/or harmonics.

The most widely used method in the stability analysis of nonlinear control systems linearized by the describing function technique is the Nyquist diagram. With the concept of the variable critical point, which depends upon the frequency and signal level at the input of nonlinearity, the Nyquist diagram permits a simple stability analysis and investigation of sustained oscillation.

However, the Nyquist diagram is inconvenient to apply to multiloop control systems. Difficulties also arise with the adjustment of the system parameter, since the diagram should be plotted each time a parameter is changed. Thus, this method can hardly be applied to design problems in which the describing function does not appear as a variable gain factor.

Other methods may be used, namely the Hurwitz criterion, Root-locus techniques, Bode diagrams, Nichols charts, Mitrovic's method, approximation method, Popov method, and phase plane techniques, etc. which can be used to investigate the nonlinearity of the system. However the parameter plane

method is more convenient since the nonlinearity values can be represented by the parameters.

Consider the sinusoidal system;

$$T(j\omega) = \frac{N(j\omega, \alpha, \beta)}{D(j\omega, \alpha, \beta)}$$

Then

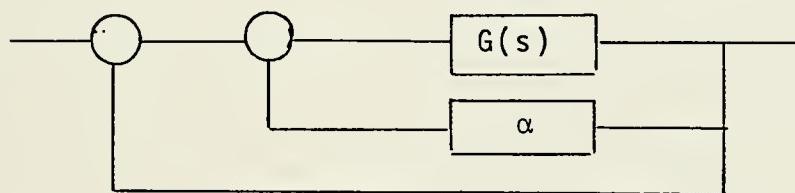
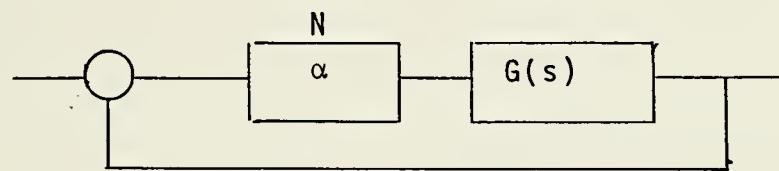
$$M = |T(j\omega)| = \left| \frac{N(j\omega, \alpha, \beta)}{D(j\omega, \alpha, \beta)} \right|$$

Define one parameter as β to be constant and the other parameter α represents the value of the nonlinear element. So;

$$M = \left| \frac{N(j\omega, \alpha)}{D(j\omega, \alpha)} \right|$$

The equation has three variables: magnitude, frequency, and the nonlinear parameter α , which the parameter plane method is to plot.

The block diagram of the nonlinear system may be the following:



α is the nonlinear element.

$G(s)$ represented the linear system.

Nonlinear elements such as square law devices, coulomb friction, dead space, saturation, backlash, relays, etc. may be represented by their describing function. The parameter is then used to represent any adjustable parameter in the describing function, and the resulting plots on the frequency parameter plane indicate the effect of the nonlinearity on the system behavior.

V. DYNAMIC SYSTEM DESIGN

A. INTRODUCTION

In the design of compensators for linear or nonlinear systems, linear system design to make the system stable uses the root locus, Bede plot, Nyquist and Nichol chart. However the parameter plane method is more convenient for both linear and nonlinear systems. In classical methods of the root locus and frequency domain (Bode diagram), the compensators are introduced to reshape the corresponding graphs of the original system in order to achieve desired system response characteristics.

In the parameter plane approach the direct effects of the additional blocks and their connection is to change the characteristic equation, so that the additional adjustable parameters are conveniently distributed among the coefficients of the characteristic equation. This enables us to have control over the characteristic roots by varying the introduced parameters.

The parameter plane is used here to investigate and design the system for antiroll stabilization of a ship by means of passive tanks. The size and properties of the tank are designed. The problem is extended by including the pump system thus converting the system to an active stabilization system.

B. THE DYNAMICS OF A SHIP IN ROLL MODE

A mathematical description of the dynamic motion of a ship requires six degrees freedom, three equations of force and three moment equations, these six equations are too complex for analytical solution. If only the rolling of the ship is considered, it may be modeled as a pure rotational motion about the longitudinal axis of ship, with no moments about the Y and Z axes.

The ship in rolling, is analogous to a pendulum. The dynamic equation of a pendulum is:

$$m \ddot{\phi} + B \dot{\phi} + \frac{m g}{l} \phi = F$$

where m = mass of the pendulum.

B = damping coefficient.

g = gravity of the earth.

l = the length of the pendulum.

ϕ = the instantaneous angle of pendulum.

F = the excite force.

Including hydrodynamic damping, the dynamic rolling equation of a ship is the same as the pendulum.

$$J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi = K_\gamma \gamma$$

where J_s = total moment of inertia around axis of roll.

B_s = hydrodynamic damping coefficient.

K_s = static righting moment coefficient.

γ = angle of drift, radians.

K_γ = drift-angle coupling coefficient.

ϕ = instantaneous angle of rolling.

The equation can be written as:

$$\ddot{\phi} + 2\zeta_s \omega_s \dot{\phi} + \omega_s^2 \phi = U_\gamma \gamma$$

where ζ_s = damping factor of the ship. $= \sqrt{\frac{B_s}{J_s K_s}}$

ω_s = natural frequency of the ship roll. $= \sqrt{\frac{K_s}{J_s}}$

$$U_\gamma = \frac{K_s}{J_s}$$

When the ship is operating in a sea state, there is a multitude of forcing frequencies present and the integrated response to all these frequencies must be considered. Further experiments are therefore carried out with the model at zero speed in irregular beam waves. The rolling of the ship is caused by the force due to the wave slope. Then the equation becomes:

$$\ddot{\phi} + 2\zeta_s \omega_s \dot{\phi} + \omega_s^2 \phi = \omega_s^2 \psi$$

where ψ = the magnitude of wave slope.

C. MODEL OF A SHIP

The model of ship used for the example, to simulate and design the tank stabilizer is one given in Reference [6], the data and characteristics of the ship were the following:

Displacement weight $w = 936$ tons.

Metacentric height $GM = 0.73$ meter.

Ship moment of inertia $J_s = 1290$ tons-m-sec².

Hydrodynamic damping coefficient $B_s = 60$ tons-m-sec.

Static righting moment coefficient $K_s = 684$ tons-m.

The mass moment of inertia of the tank stabilizer (J_t) increased the value of J_s , but it is very small and can be considered negligible when compared to J_s , for convenience use the value of J_s as above.

The beam of the ship can be found by,

$$B = \frac{2\pi\sqrt{GM}}{C \sqrt{\frac{K_s}{J_s}}} = \frac{2\pi\sqrt{GM}}{C \omega_s}$$

where C = a coefficient dependent upon the moment of roll and approximate is $0.797 \text{ sec/m}^{\frac{1}{2}}$.

Therefore the beam of the ship can be calculated and the result is:

$$B = 9.25 \text{ meters.}$$

D. PASSIVE TANK STABILIZATION

1. Analysis

A passive tank can only be designed for optimum performance for one ship condition of displacement and metacentric height. When the ship condition varies, the free surface correction μ varies, together with the quantity of water required. It is therefore necessary to check the performance of the stabilizer under different ship loading conditions and it could be that a compromise design has to be adopted which is acceptable for all conditions.

The free surface loss is generally expressed as:

$$\frac{i}{V} = \frac{5 w B}{48 \Delta h}$$

where i = the second moment of inertia of the free surface.

V = the volume of displacement of the ship.

w = the weight of active fluid.

Δ = displacement of the ship.

B = beam of the ship.

h = depth of fluid in tank.

and $\frac{i}{V}$ = usual notation for free surface correction.

The tank size parameter is limited primarily by static stability requirements. However, the tank should be designed with as large a free surface as possible consistent with the requirements of static stability. Choosing values of μ is equivalent to varying the tank size.

Since the pendulum system is analogous to ship rolling, the effect on the ship of a hydrodynamic tank stabilizer is analogous to the addition of a second pendulum. The combination of two pendulums in series, may be called a double-pendulum system as shown in Figure 5.1.

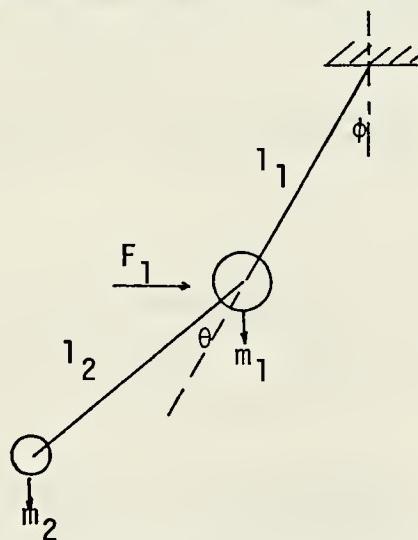
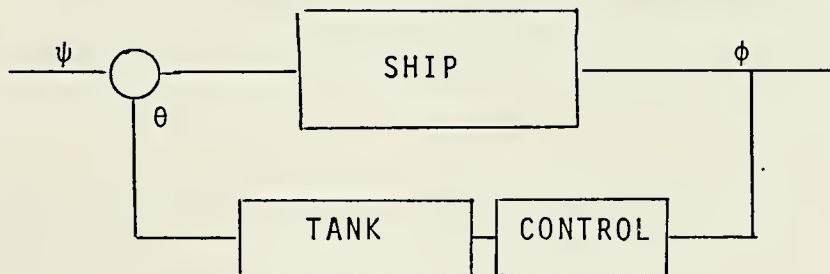


Figure 5.1 Double-Pendulum System.

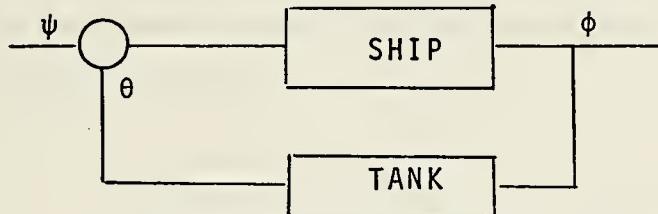
The first pendulum represents the ship rolling system, and the second pendulum represents the dynamic tank system. The length of the pendulums should be the same, so that the periods are nearly equal which means that both natural frequencies of the ship rolling and the tank are approximately the same. When the ship mass has been swung, the free oscillations are observed. When the tank mass has been attached, the masses of both ship and tank can be swung and the interplay and exchange of energy between the tank and the ship can be seen.

The tank stabilization system uses water or some fluid mass device. There are many types of tanks. They can be designed as Doughnut tanks, completely filled sea ducted tanks, partially filled sea ducted tanks, U-tube tanks and free surface tanks. These tanks systems were suggested by Dr. J. H. Chadwick Jr. Typical tanks such as free surface and U-tube tanks have been suggested by G. J. Goodrich and Williams C. Webster respectively. Both types are described and designed in this chapter.

A block diagram of ship and tank system is:



For a passive tank system the "Control block" does not exist, or may be considered a unit gain. Thus the block diagram for a passive system becomes:



The relationship of the ship with a tank stabilizer can be represented as a double pendulum. The equations are:

$$J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi + J_{st} \ddot{\theta} + K_{st} \theta = F_1$$

$$J_{st} \ddot{\phi} + K_{st} \phi + J_t \ddot{\theta} + B_t \dot{\theta} + K_t \theta = 0$$

where J_s = the moment of inertia of the ship rolling.
 B_s = hydrodynamic damping coefficient of ship.
 K_s = the static righting moment coefficient of ship.

J_t = the moment of inertia of the tank.

B_t = the damping coefficient of the tank.

K_t = the static equivalent coefficient of the tank.

J_{st} = the coupling moment of inertia of ship/tank.

K_{st} = the coupling static coefficient of ship/tank.

F_1 = the force insert in ship as the wave slope.

2. Free Surface Tank Design

The free surface tank as Figure 5.2 shown, is advantageous in decreasing the magnitude gain of rolling, and also decreases stability of the ship since the free surface loss is high.

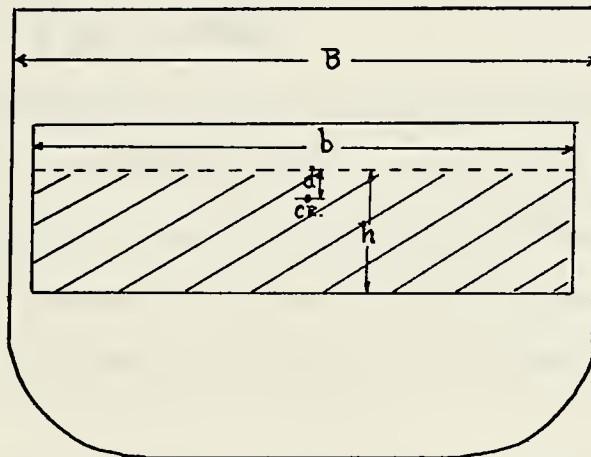


Figure 5.2 Free Surface Tank.

The equations of the ship and tank can be rewritten as: for the ship

$$\ddot{\phi} + 2\zeta_s \omega_s \dot{\phi} + \omega_s^2 \phi + \mu \frac{d}{g} \omega_s^2 \theta - \mu \omega_s^2 = \omega_s^2 \psi$$

for the tank

$$\frac{d}{g} \omega_t^2 \ddot{\phi} - \omega_t^2 \phi + \ddot{\theta} + 2\zeta_t \omega_t \dot{\theta} + \omega_t^2 \theta = 0$$

where ω_s = natural frequency of the ship roll. $= \sqrt{\frac{K_s}{J_s}}$

ω_t = natural frequency of the tank. $= \sqrt{\frac{K_t}{J_t}}$

ζ_s = damping factor of the ship roll.

ζ_t = damping factor of the tank.

g = gravity of the earth = 9.8 m/sec.²

d = the distance between the water level in the tank and the roll center.

ψ = wave slope function.

μ = the free surface correction factor.

$$= \frac{i}{\nabla GM} = \frac{1}{2\Delta GM} b^2$$

Both equations have the negative sign in the coupling terms since the restoring forces are in opposite directions. If the distance d is positive which means the level of water in the tank is higher than the rolling center, the magnitude gain is less as shown in Figure 5.3, so for convenience set d equal zero.

Generally, the period of rolling of the tank (or natural frequency) must be designed nearly equal to the period of the rolling of the ship. The simplified equations become:

$$\ddot{\phi} + \alpha_s \dot{\phi} + \omega_n^2 \phi - \mu \omega_n^2 \theta = \omega_n^2 \psi$$

$$\ddot{\theta} + \alpha_t \dot{\theta} + \omega_n^2 \theta - \omega_t^2 \phi = 0$$

where $\alpha_s = 2\zeta_s \omega_s$

$$\alpha_t = 2\zeta_t \omega_t$$

$$\omega_n = \omega_s = \omega_t$$

By manipulation and transform, the system transmission function of the rolling angle due to the wave slope force is:

$$\frac{\phi}{\psi}(s) = \frac{\omega_n^2(s^2 + \alpha_t s + \omega_n^2)}{s^4 + (\alpha_t + \alpha_s)s^3 + (2\omega_n^2 + \alpha_t \alpha_s)s^2 + \omega_n^2(\alpha_t + \alpha_s)s + \omega_n^4(1-\mu)}$$

The solution of the equation can be expressed as an amplification factor $A(\omega)$ and ϵ the phase angle lag of the forcing function.

$$A(\omega) = \frac{\omega_n^2}{\sqrt{x^2 + y^2}}$$

$$\epsilon = \tan^{-1} \frac{y}{x}$$

$$\text{where } x = (\omega_n^2 - \omega^2) - \frac{\mu \omega_n^4 (\omega_n^2 - \omega^2)}{(\omega_n^2 - \omega^2)^2 + \alpha_t^2 \omega^2}$$

$$y = \omega [\alpha_s + \frac{\mu \alpha_t \omega_n^4}{(\omega_n^2 - \omega^2)^2 + \alpha_t^2 \omega^2}]$$

These solutions can be used to investigate the effect of variation of the tank parameters α_t and μ . Calculations can be carried out using variation of each parameter in turn, keeping the other constant. Adjust both parameters to obtain the minimum magnitude gain. These values of parameters are used to design the tank stabilizer.

The computer simulation program used is due to Glavis [7]. The data of the ship in the previous section were substituted into the equation of ship and tank stabilizer, two unknown parameters in the equation are α_t and μ as variables.

Recall

$$\frac{\phi}{\psi}(s) = T(s) = \frac{\omega_n^2(s^2 + \alpha_t s + \omega_n^2)}{s^4 + (\alpha_t + \alpha_s)s^3 + (2\omega_n^2 + \alpha_t \alpha_s)s^2 + \omega_n^2(\alpha_t + \alpha_s)s + \omega_n^2(1-\mu)}$$

Numbering the data of the ship and parameters the equation becomes:

$$T(s) = \frac{0.53023s^2 + 0.53023\alpha_t s + 0.28115}{s^4 + (0.0465 + \alpha_t)s^3 + (1.06047 + 0.0465\alpha_t)s^2 + (0.02466 + \\ 0.53023\alpha_t)s + 0.28115(1-\mu)}$$

For the sinusoidal signal the equation becomes:

$$T(j\omega) = \frac{0.28115 - 0.53023\omega^2 + j0.53023\alpha_t\omega}{\omega^4 - (1.06047 + 0.04651\alpha_t)\omega^2 + 0.28115(1-\mu) + j[(0.024662 \\ + 0.53023\alpha_t)\omega - (0.0465 + \alpha_t)\omega^3]}$$

Then

$$M^2 = \frac{(0.28115 - 0.53023\omega^2)^2 + (0.53023\alpha_t\omega)^2}{[\omega^4 - (1.06047 + 0.04651\alpha_t)\omega^2 + 0.28115(1-\mu)]^2 + [(0.024662 \\ + 0.53023\alpha_t)\omega - (0.0465 + \alpha_t)\omega^3]}$$

Since the size of the tank depends on the free surface coefficient (μ), if μ is large, the tank will be big and the stability of the ship will be decreased because the metacentric height decreases. So the magnitude gain of rolling in the region of the natural frequency decreases but at low frequency the magnitude gain increases.

Consider the computer results. Figure 5.4 shows curves of the tank damping coefficient versus frequency, with magnitude as curve index and the free surface coefficient is constant. On Figure 5.4 the free surface coefficient is 0.05. If α (tank damping coefficient factor) is approximately 0.25 the height of the resonance peak is 13 db, which is the

smallest value obtainable for these conditions. This occurs at a frequency of about 0.72 rad/sec. At a frequency of about 0.18 rad/sec, the magnitude is only 1 db, and is even smaller at low frequency.

On Figure 5.5, the free surface coefficient is 0.1, the line of constant tank damping coefficient factor is 0.35 to give a resonance peak of about 11 db which is the minimum obtainable. Note, however, that at lower frequencies the magnitude is higher than for the condition of Figure 5.4.

If the free surface is set equal to 0.2 as on Figure 5.6, the tank damping coefficient should be 0.45, which gives 9 db for the resonance peak and 2 db at low frequency (0.5 rad/sec). Thus, if the free surface coefficient is increased, the magnitude in the region of low frequency also is increased, though the height of the resonance peak is decreased.

The curves of Figure 5.7, Figure 5.8 and Figure 5.9 give tank damping coefficient factor versus magnitude. Frequency is the curve index, and the free surface coefficients are 0.05, 0.1 and 0.2 respectively. These Figures correspond to Figure 5.4, Figure 5.5 and Figure 5.6. By inspection they predict the same optimum values for the damping coefficient factor.

Figure 5.10, Figure 5.11 and Figure 5.12 give the Bode diagrams (plot of the magnitude versus frequency) with the tank damping coefficient factor as curve index and the

free surface coefficient factor is constant. In this case the Bode diagram cannot show the best value of the tank damping coefficient factor unless that value has been plotted. However the Bode diagram is good for consideration of the values of the free surface coefficient for which the curves show different magnitudes at all frequencies.

By inspection of the curve families the best values of the tank damping coefficient factor and the free surface coefficient factor are:

the tank damping coefficient factor = 0.25

the free surface coefficient factor = 0.05

and/or

the tank damping coefficient factor = 0.35

the free surface coefficient factor = 0.1

Note that the magnitude gain value for low frequencies increases with increases of the free surface (μ), which means increasing tank size. This results from the losses in roll stiffness due to the tank free surface effect. It is also obvious that the roll response at low frequencies increases as tank size increases.

For the free surface tank (as reference [8]), the relation between the parameters and the tank size is:

$$\mu = \frac{l b^3}{2\Delta GM}$$

where l = the length of the tank.

b = the width of the tank can be equal or less than the beam of the ship.

Δ = the displacement of the ship in tons.

GM = metacentric height

The other suggestion equation is:

$$h = \frac{\omega_t^2 b}{\pi^2 g}$$

where h = the depth of the water in tank.

If the width of the tank is assumed to be equal to the beam of ship, the depth of the water in the tank is equal to 0.5 meter. If the selected free surface coefficient factor is 0.1.

$$l = 0.82 \text{ meter.}$$

If the free surface coefficient factor is equal to 0.05, the length of the tank is reduced to:

$$l = 0.41 \text{ meter}$$

which can save space in the ship. And the other parameters such as J_t , B_t , and K_t can be calculated.

The time response to an initial condition is shown in Figure 5.13, for $\mu = 0.0, 0.05, 0.1, \text{ and } 0.2$, and $\alpha_t = 0.0, 0.25, 0.35 \text{ and } 0.45$ respectively. The curve for $\mu = 0.0$, without antiroll tank stabilizer is very oscillatory, and is damped only by the damping of the ship itself. The other curves include the tank stabilizer, so the damping is higher, and the curves reach steady state sooner. The time constant of the undamped system is about 42.5 seconds. With stabilizer added the setting time can be halved.

Figure 5.14 shows the time response to a sinusoidal input. The parameters have been varied as for Figure 5.13. The frequency of the input is set at about the natural frequency (0.728 rad/sec). For the ship without tank stabilizer the magnification is seen to be excessively large. When the tanks are added the system response is seen to be greatly reduced in magnitude.

3. U-Tube Tank Design

The U-Tude tank as designed in this chapter is assumed to be a rectangular tank. The mutual inertia term, J_{st} , which has been set to zero in preceding sections, is not necessarily zero. The value of J_{st} depends on the acceleration of the water relative to the tank, and this depends on the location of the tank. If the cross-duct passes below the center of rotation, J_{st} is a positive number which becomes smaller as the tank system is placed higher, as shown in Figure 5.15a and b.

Consider that the two parameters J_t and J_{st} are given by:

$$J_t = \rho A_0 R^2 \int_0^S \frac{A_0}{A_s} ds$$

$$J_{st} = \rho A_0 R^2 \int_0^S \frac{D_s}{R} ds$$

where ρ = mass density of the fluid in tank.

A_0 = cross-sectional area of vertical leg of the tank.

R = lever arm of the tank.

D_s = perpendicular distance from "virtual" center of rotation to the tangent at any point on the trajectory.

A_s = the cross-sectional area at any point S.

These two equations can be related as:

$$\begin{aligned}\frac{J_{st}}{J_t} &= \int_0^S \frac{D_s A_s}{R A_0} ds \\ &= 2h + \frac{2 D_s A_s}{A_0}\end{aligned}$$

and h = the height of the water in the tank.

Note that D_s can be negative if the tank is placed above the center of rotation. So J_{st} can be zero if:

$$-D_s = \frac{h A_0}{A_s}$$

Recall

$$J_s \ddot{\phi} + B_s \dot{\phi} + K_s \ddot{\phi} + J_{st} \ddot{\theta} + K_{st} \theta = K_{ss} \psi$$

$$J_{st} \ddot{\phi} + K_{st} \dot{\phi} + J_t \ddot{\theta} + B_t \theta + K_t \dot{\theta} = 0$$

By using the definition as before the transformed equations become.

$$(S^2 + \alpha_s S + \omega_n^2) \phi + \left(-\frac{J_{st}}{J_s} S^2 - \frac{K_t}{J_s} \right) \theta = \omega_n^2 \psi$$

$$\left(-\frac{J_{st}}{J_t} S^2 - \omega_n^2 \right) \phi + (S^2 + \alpha_t S + \omega_n^2) \theta = 0$$

Define $\frac{K_t}{J_s} = \beta$ ($= \mu \omega_s^2$ as in the free surface tank)

$$\frac{J_{st}}{J_t} = X = 2h + \frac{2 D_s A_s}{A_0}$$

$$\alpha_t = \alpha$$

then $\frac{J_{st}}{J_s} = \frac{\beta X}{\omega_n^2}$

Using Cramer's rule, the ship roll transfer function due to the wave slope force is:

$$\frac{\phi}{\psi}(s) = \frac{\omega_n^2(s^2 + \alpha_t s + \omega_n^2)}{(1 - \frac{X^2 \beta}{\omega_n^2})s^4 + (\alpha_s + \alpha_t)s^3 + (2\omega_n^2 + \alpha_t \alpha_s - 2X\beta)s^2 + \omega_n^2(\alpha_t + \alpha_s)s + \omega_n^4 \beta \omega_n^2}$$

Inserting the values of the parameters of the ship, the equation becomes:

$$\frac{\phi}{\psi}(s) = \frac{0.53023s^2 + 0.53023\alpha s + 0.28115}{[(1 - 1.886X^2 \beta)\omega_n^4 + (0.0465 + \alpha)s^3 + (1.06047 + 0.0465 - 2X\beta)s^2 + (0.02466 + 0.53023\alpha)s + 0.28115 - 0.53023\beta]}$$

then

$$M^2 = \frac{(0.28115 - 0.53023\omega^2)^2 + (0.53023\alpha\omega)^2}{[(1 - 1.886X^2 \beta)\omega^4 - (1.06047 + 0.0465 - 2X\beta)\omega^2 + 0.28115 - 0.53023\beta]^2 + [(0.024662 + 0.53023\alpha)\omega - (0.0465 + \alpha)\omega^3]^2}$$

Figure 5.16a, b and c are plots for $X=3$ and $\beta=0.05$, there are two resonance peaks in the magnitude curve, one for the ship itself and the other for the tank. The value of α at the minimum peak magnitude is about 0.45 and only one peak magnitude is obtained. The point P in Figure 5.16c corresponds to the line of -2 db in Figure 5.16a.

If the tank is placed as high as possible, the value of X may be zero or negative. If $X=0$, the result is the same as for the free surface tank with $\beta=\mu\omega_n^2$. The plot of $X=-1.0$ and $\beta=0.05$ is shown as Figure 5.17a, b, and c. The points P and Q on Figure 5.17c are the intersection of the α lines, these points correspond to the lines of the frequencies 0.6 and 0.9 rad/sec on Figure 5.17a. In this frequency region, if the value of α is decreased, the magnitude also decreases. Figure 5.17b shows that minimum peak value of magnitude is obtained with $\alpha=0.7$.

The U-Tube tank design equation is:

$$K_t = 2\rho g A_0 R^2$$

where ρ = mass density of the water.

g = gravity of the earth.

A_0 = cross-sectional area of each vertical leg of the tank.

R = distance from ship center line to center of a tank vertical leg.

The product of ρ and g is unity for the water, so both can be eliminated from the equation.

And the cross-sectional area of the channel between tanks is given by:

$$A_s = \frac{A_0 R \omega_s^2}{g - h \omega_s^2}$$

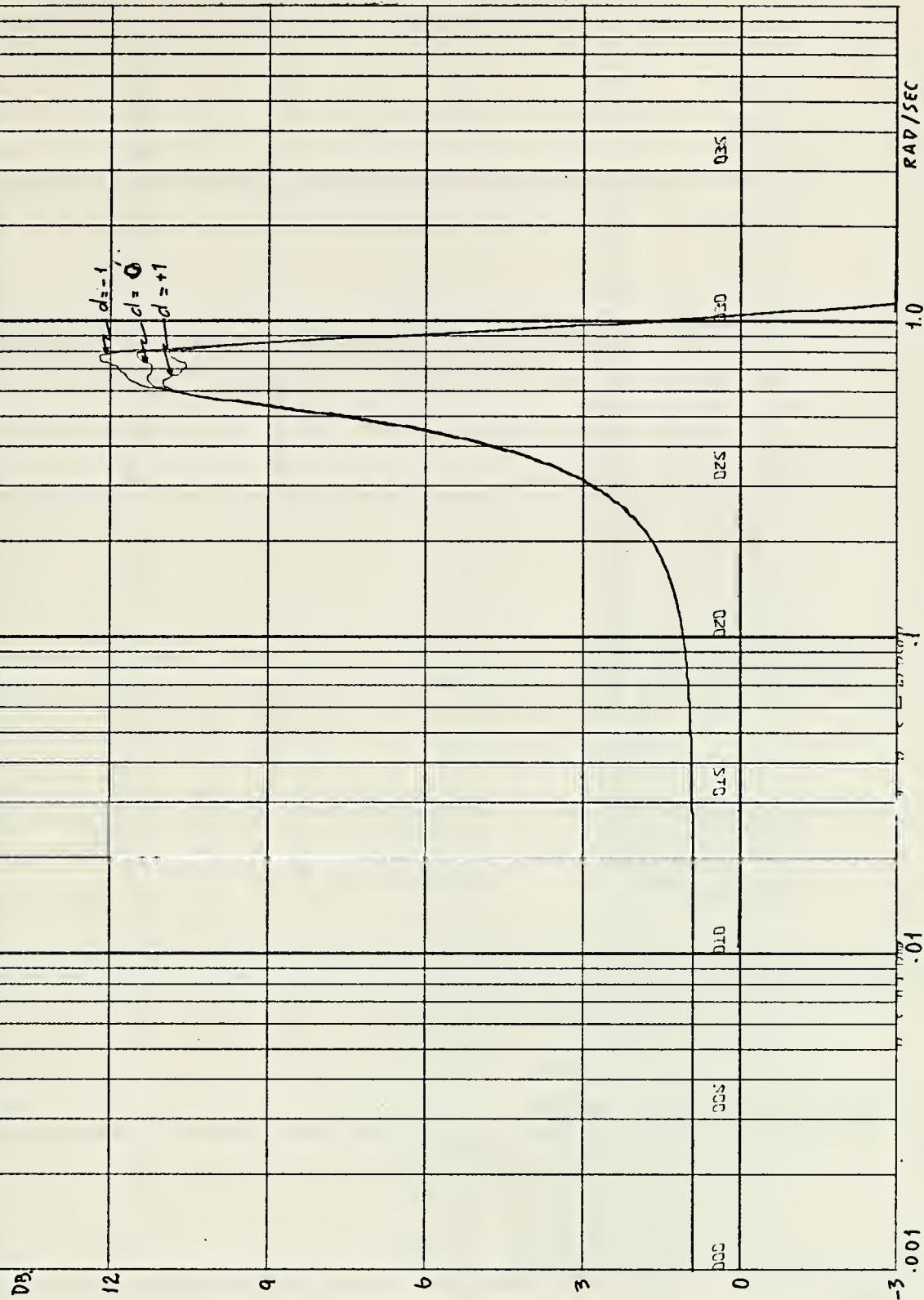


Figure 5.3 Frequency Parameter Plane (Bode Diagram). Plot of the effect of the water level in Passive free-surface tank.

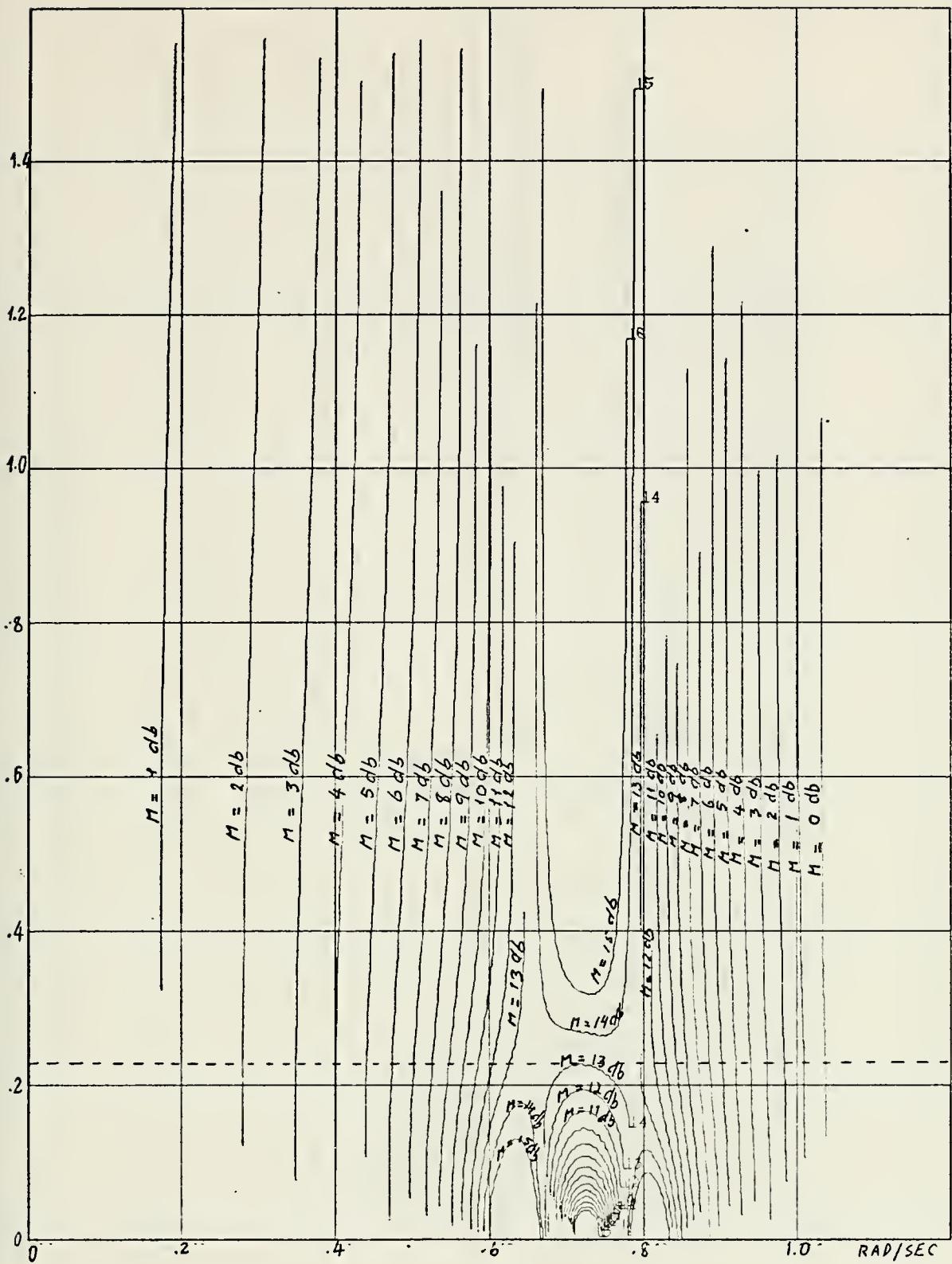


Figure 5.4 Frequency Parameter Plane for Free-surface Tank.
 Plot of Tank Damping Factor vs Frequency with
 Free-surface, $\mu=0.05$ and M as curve index.

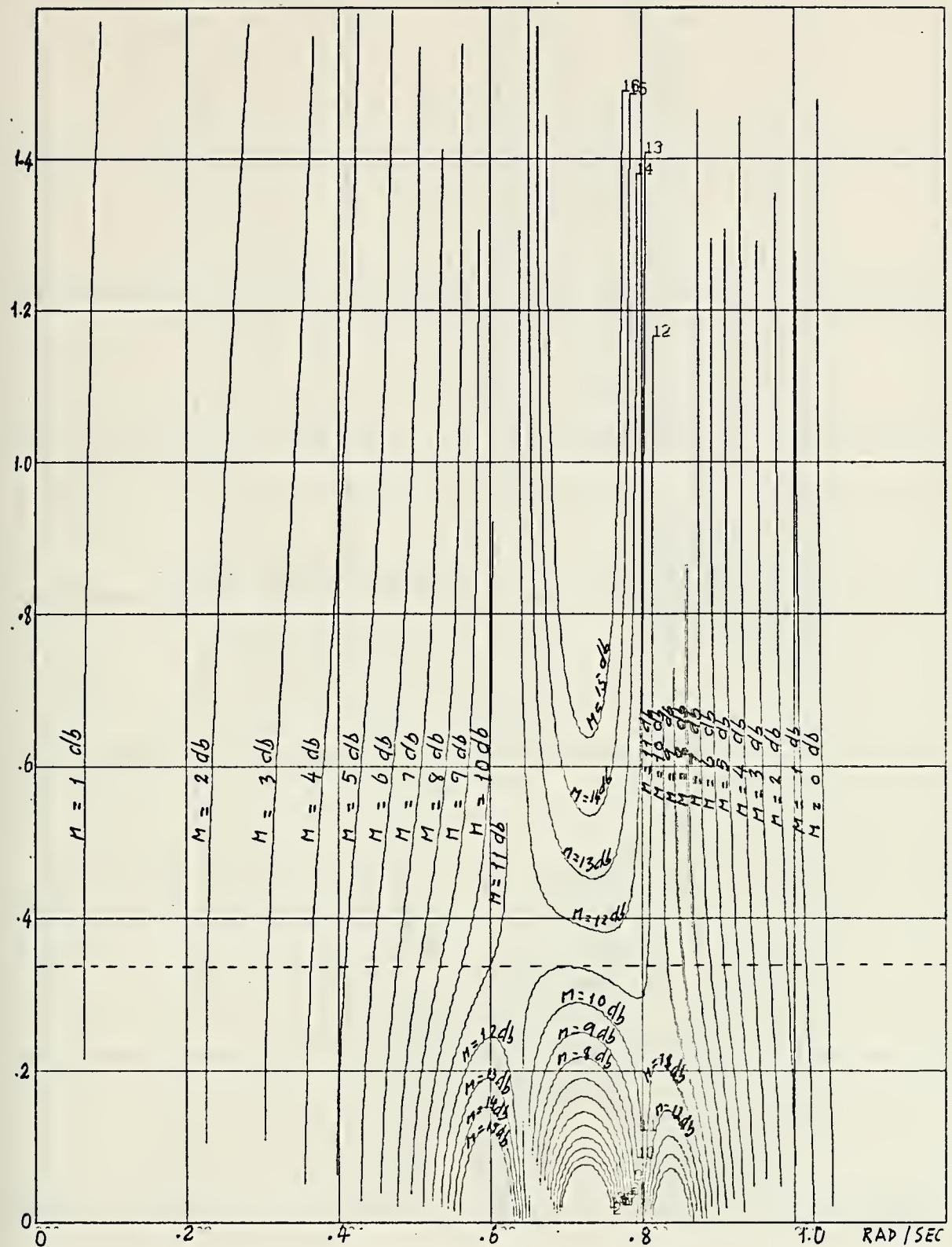


Figure 5.5 Frequency Parameter Plane for Free-surface Tank.
Plot of Tank Damping Factor vs Frequency with
Free-surface, $\mu=0.1$ and M as curve index.

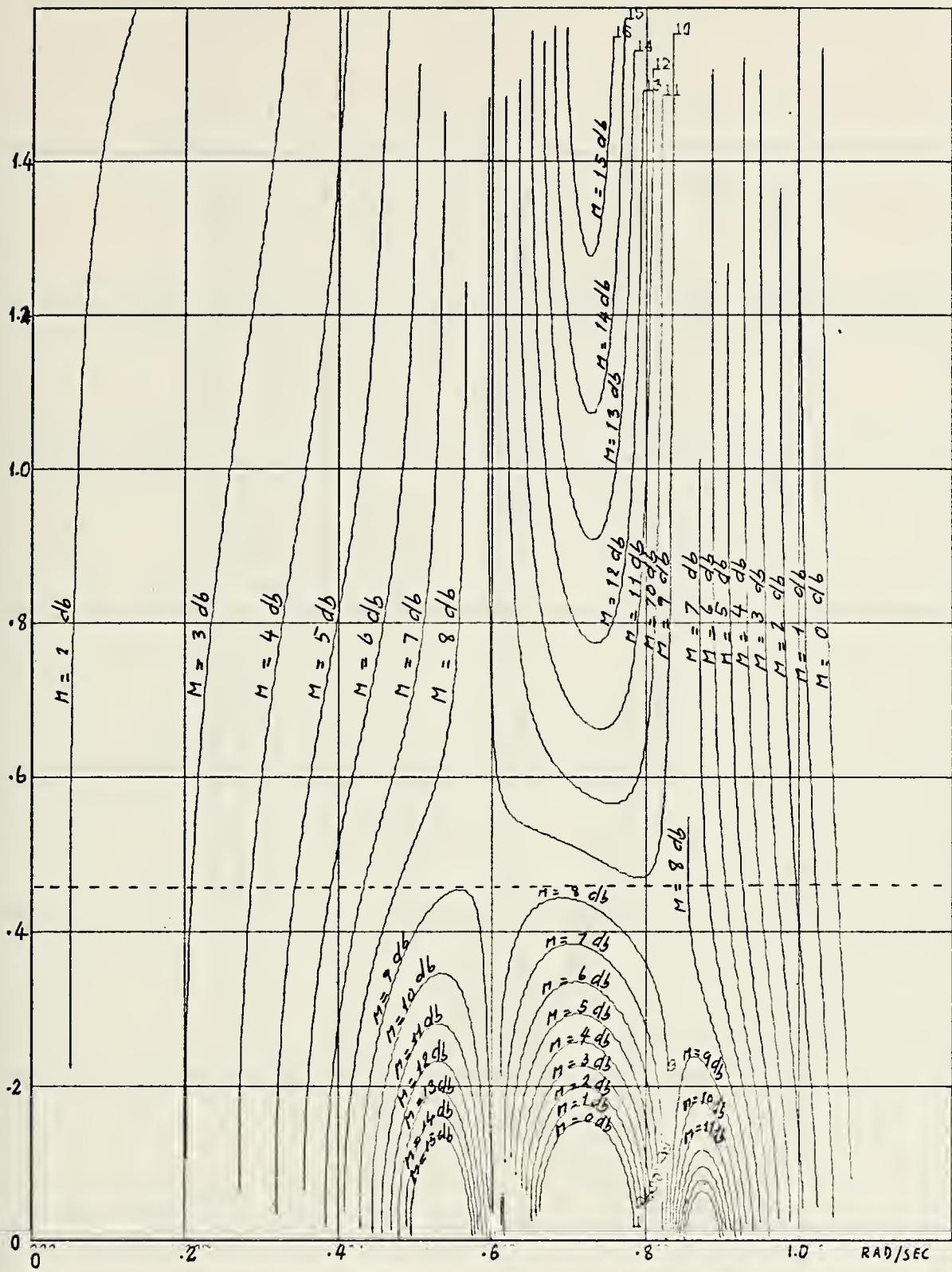


Figure 5.6 Frequency Parameter Plane for Free-surface Tank.
Plot of Tank Damping Factor vs Frequency with
Free-surface, $\mu=0.2$ and M as curve index.

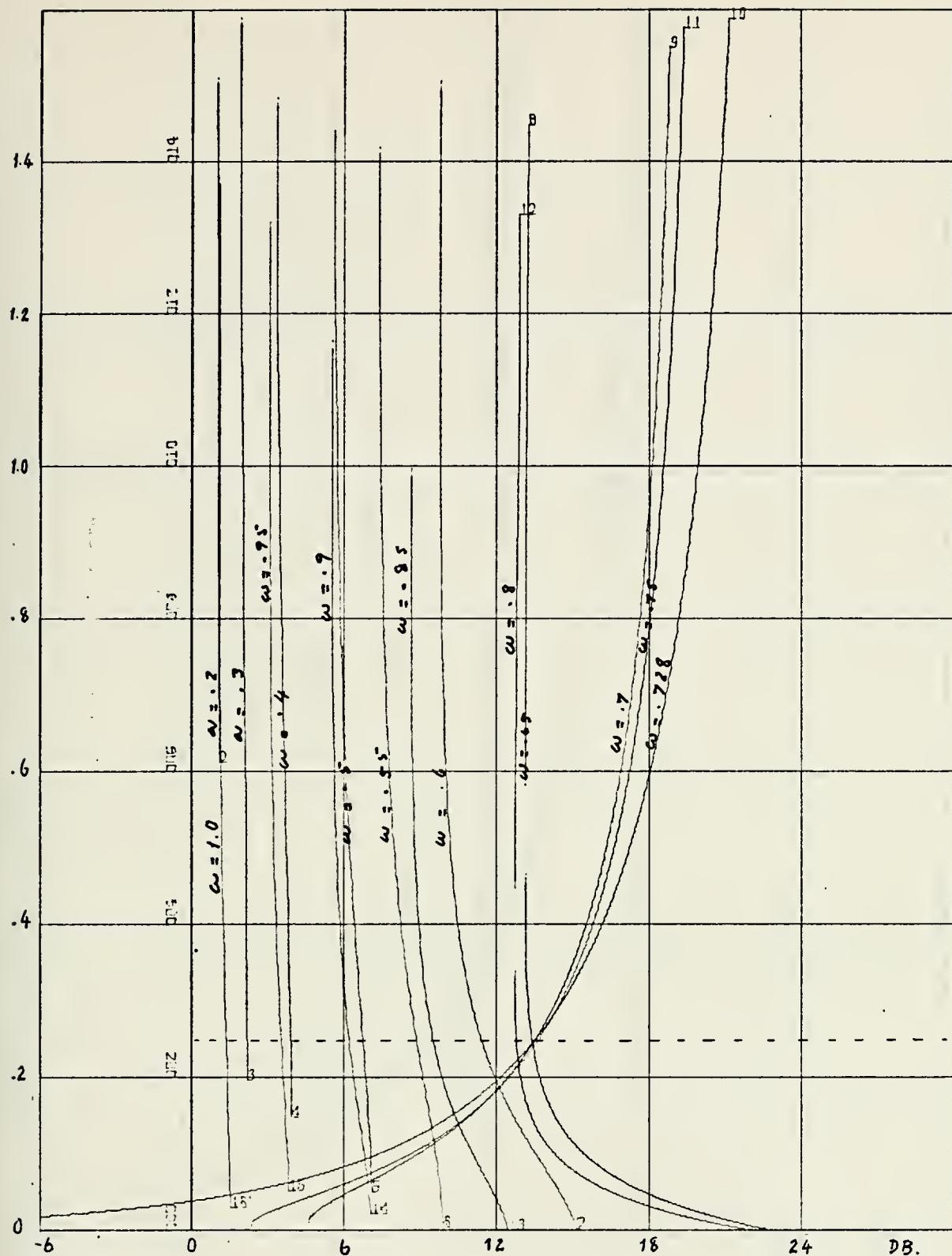


Figure 5.7 Frequency Parameter Plane for Free-surface Tank.
Plot of Tank Damping Factor vs Magnitude with
Free-surface, $\mu=0.05$ and ω as curve index.

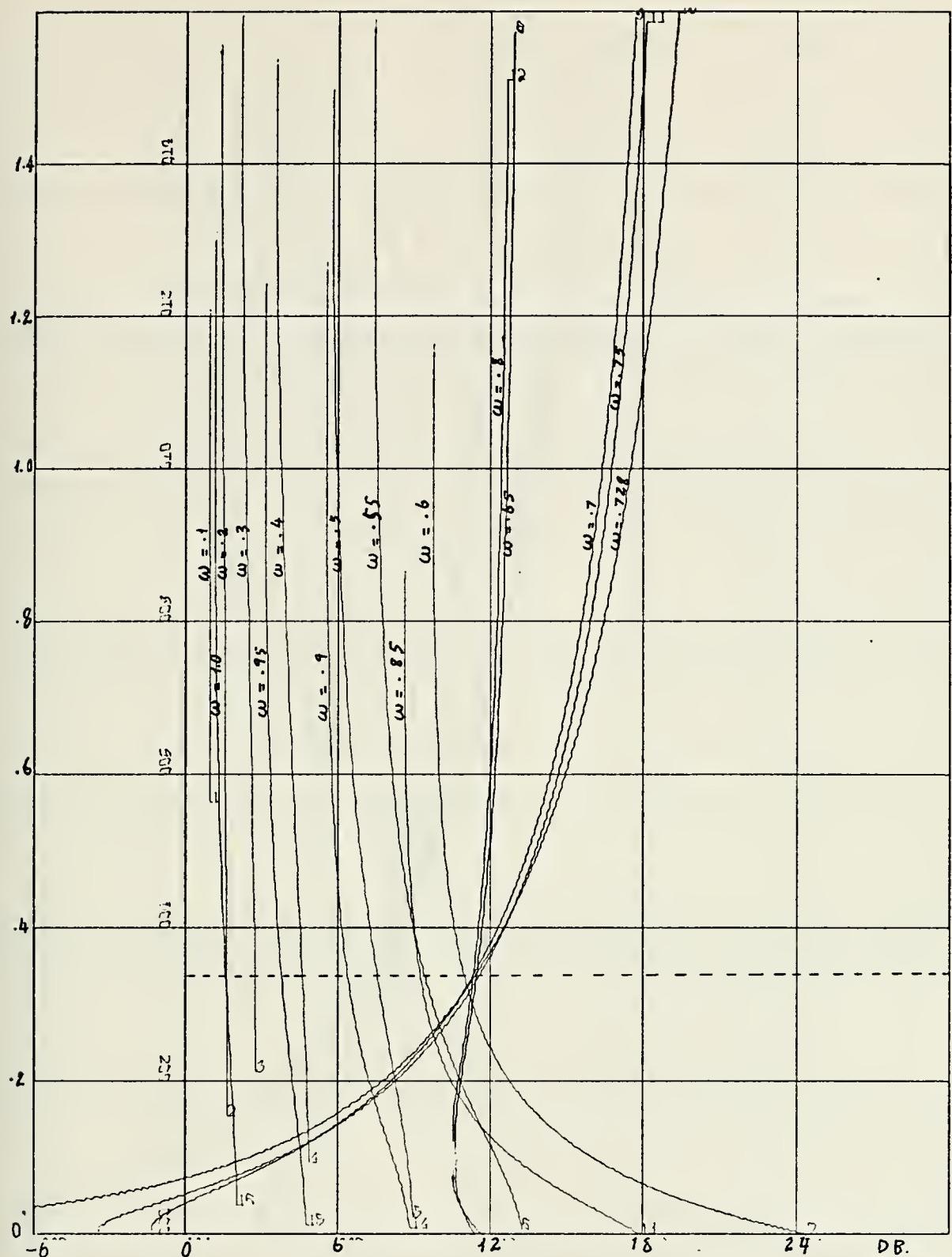


Figure 5.8 Frequency Parameter Plane for Free-surface Tank.
Plot of Tank Damping Factor vs Magnitude with
Free-surface, $\mu=0.1$ and ω as curve index.

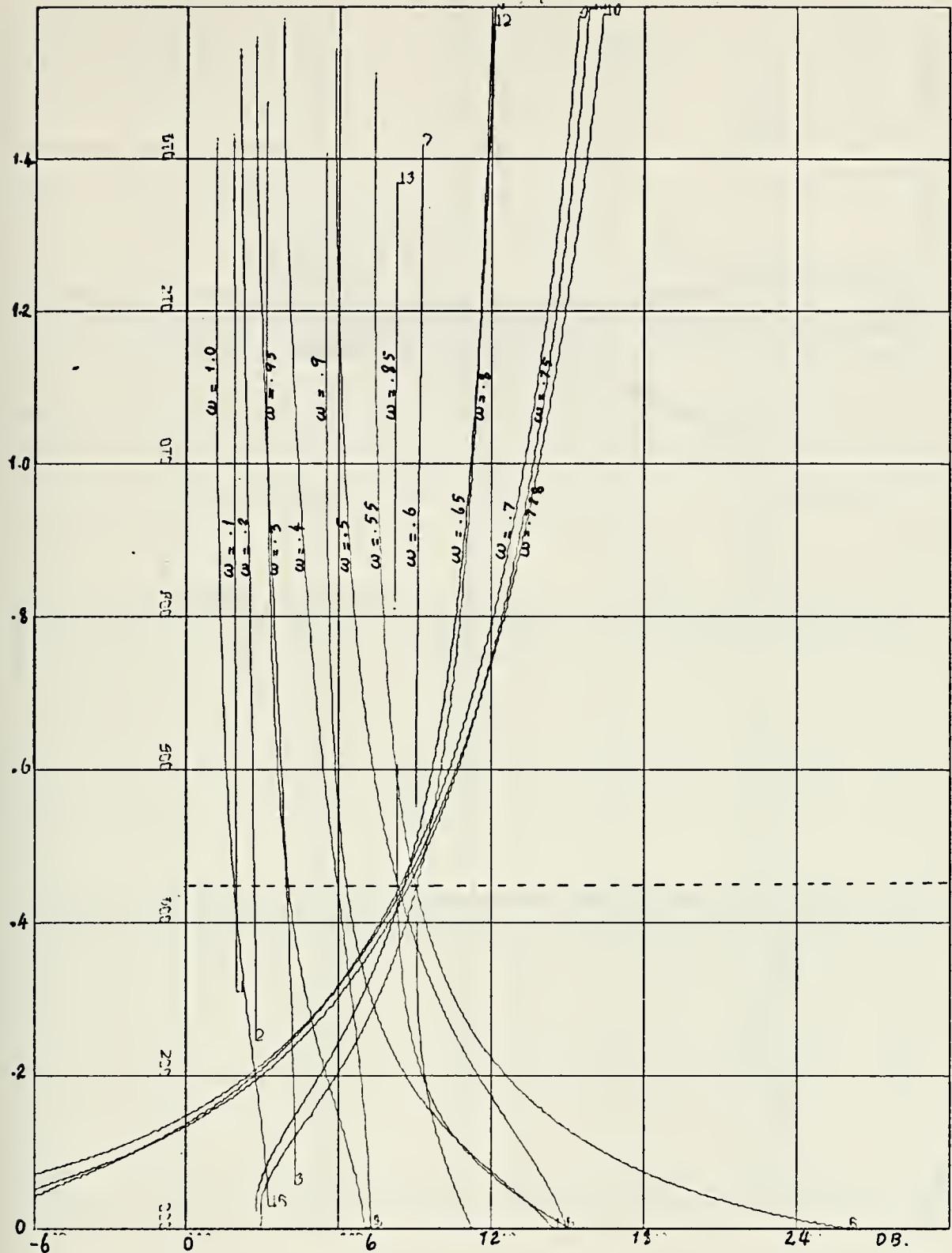


Figure 5.9 Frequency Parameter Plane for Free-surface Tank.
Plot of Tank Damping Factor vs Magnitude with
Free Surface, $\mu = 0.2$ and ω as curve index.

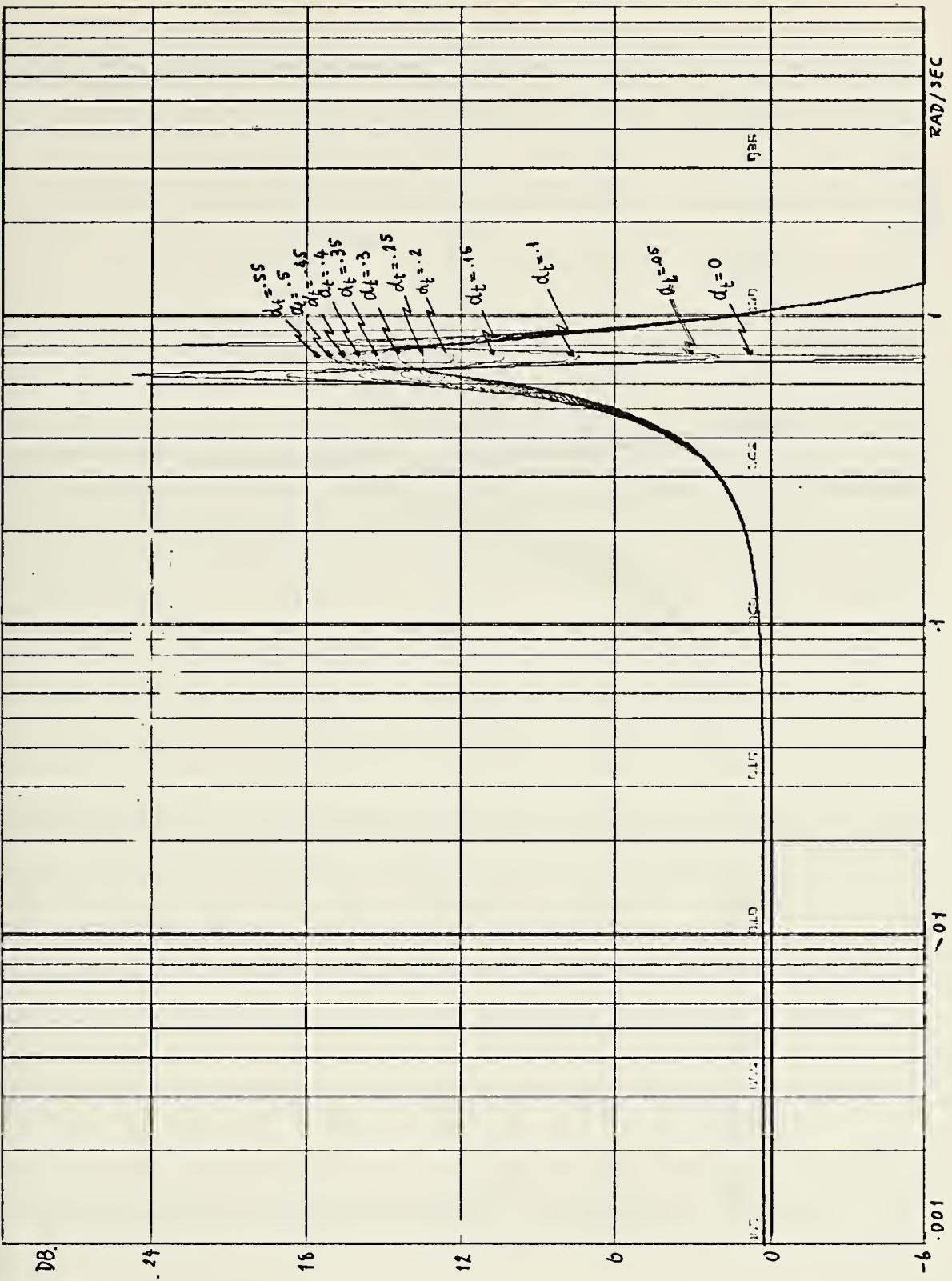


Figure 5.10 Frequency Parameter Plane for Free-surface Tank.
Bode Diagram, Plot of Tank Damping Factor as
curve Index with Free-Surface, $\mu=0.05$.

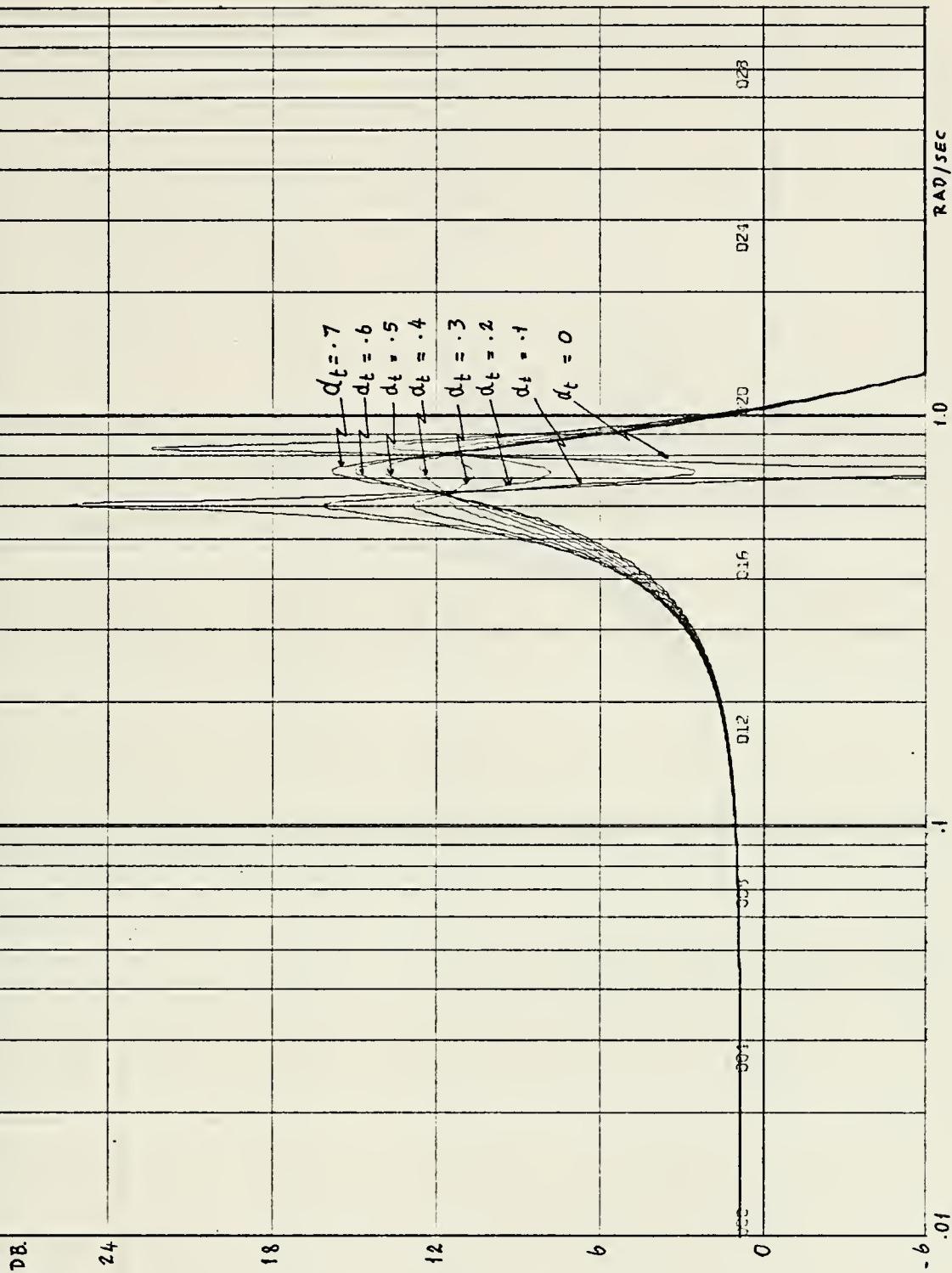


Figure 5.11 Frequency Parameter Plane for Free-surface Tank.
Bode Diagram, Plot of Tank Damping Factor as
Curve Index and Free-surface, $\mu=0.1$.

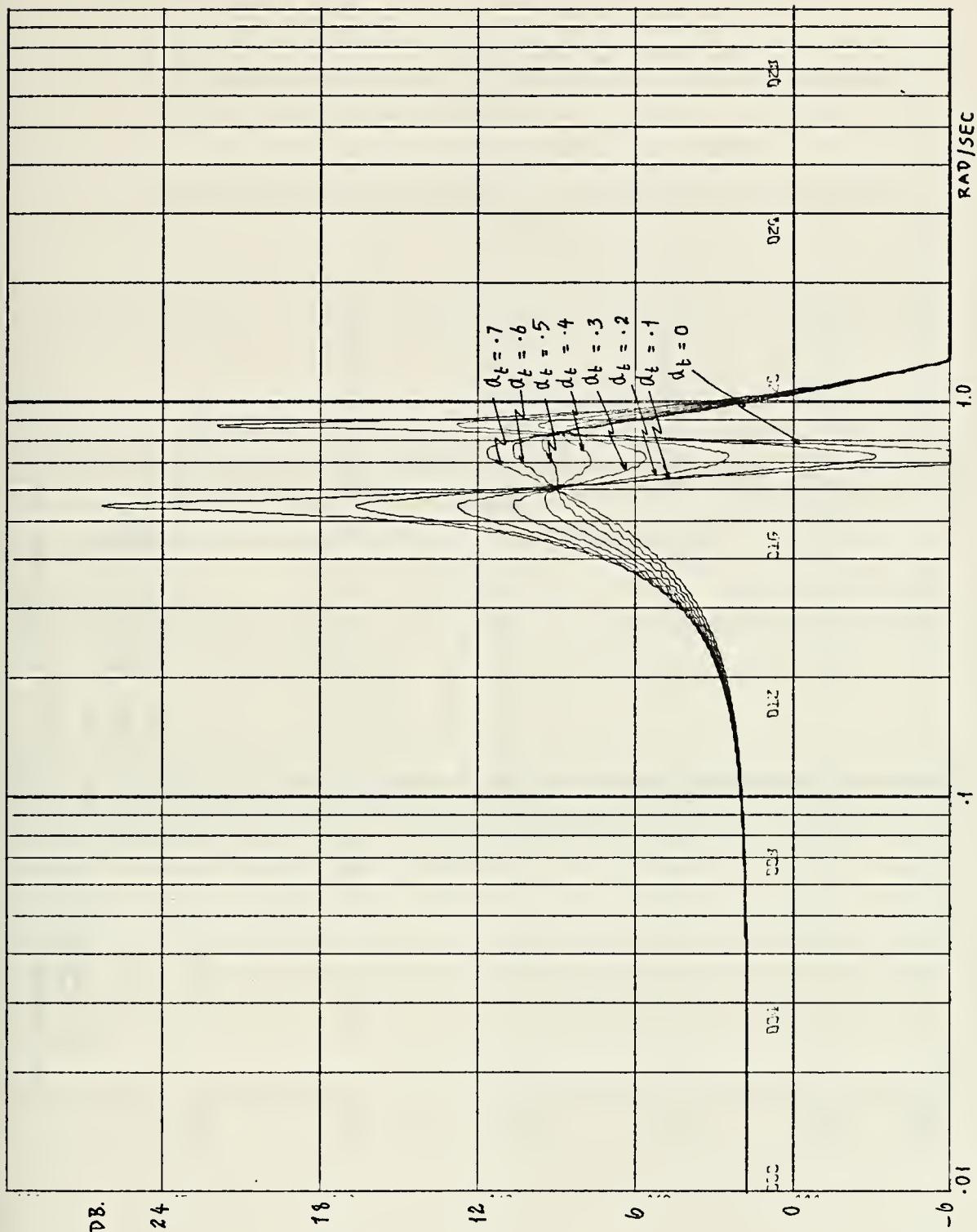


Figure 5.12 Frequency Parameter Plane for Free-surface Tank.
Bode Diagram, Plot of Tank Damping Factor as
Curve Index and Free-surface, $\mu=0.2$.

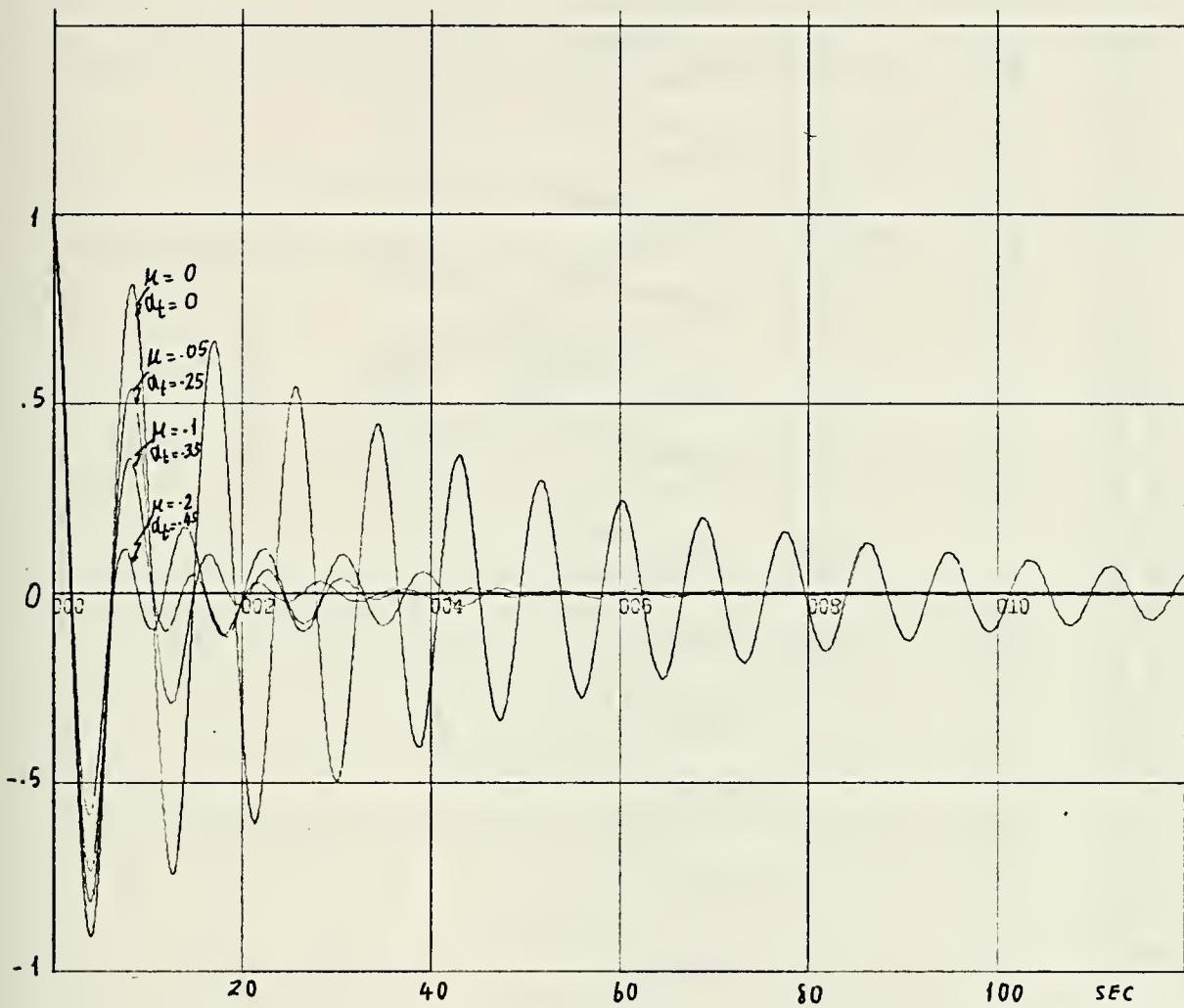


Figure 5.13 Initial Condition Response with Different Value of Tank Damping Factor and Free-surface.

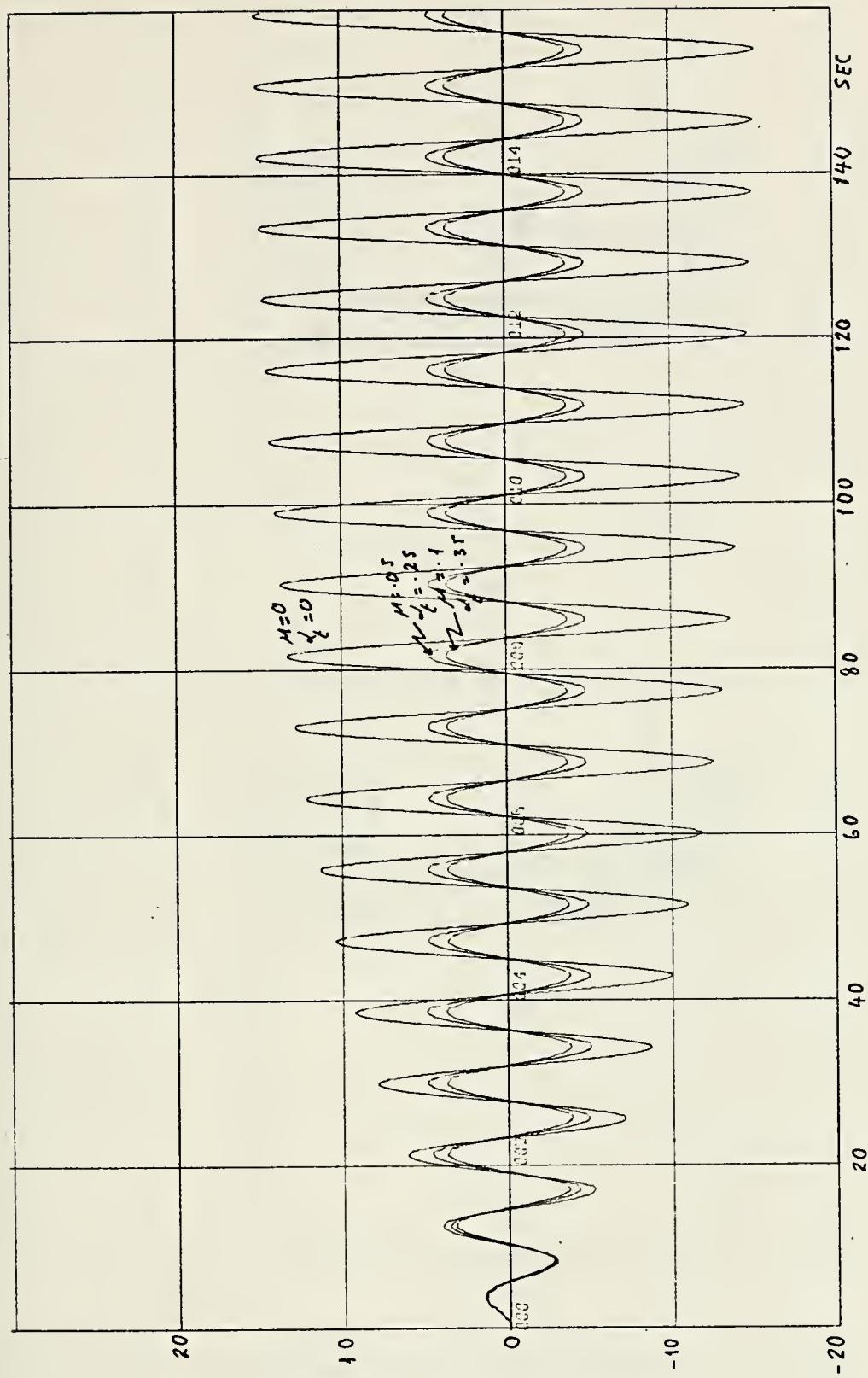
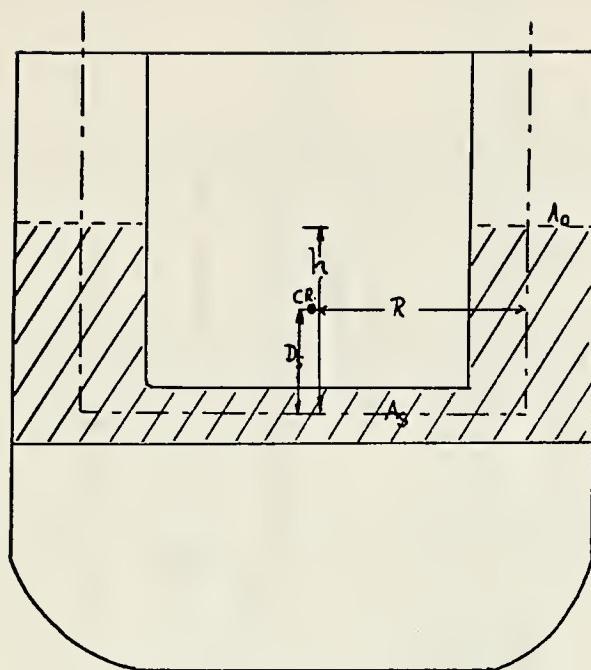
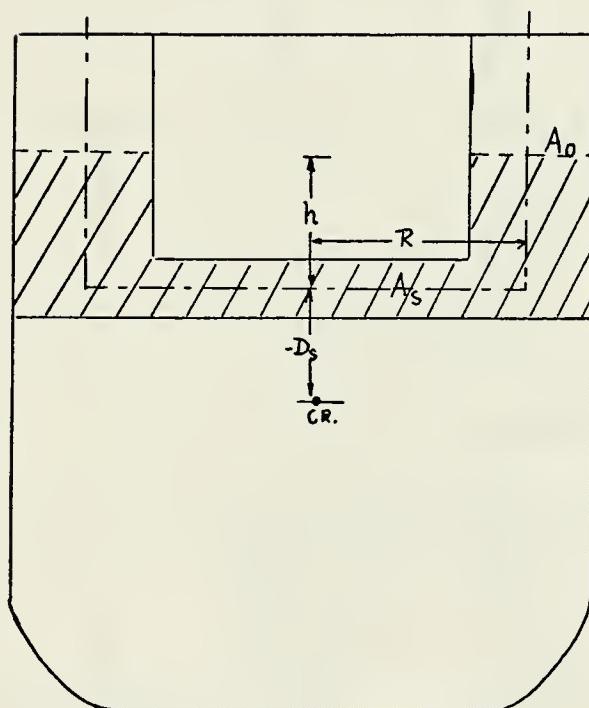


Figure 5.14 Sinusoidal Response ($\omega = .728 \text{ RAD/SEC}$) with Different Value of Tank Damping Factor and Free surface.



(a)



(b)

Figure 5.15 U-Tube Tank

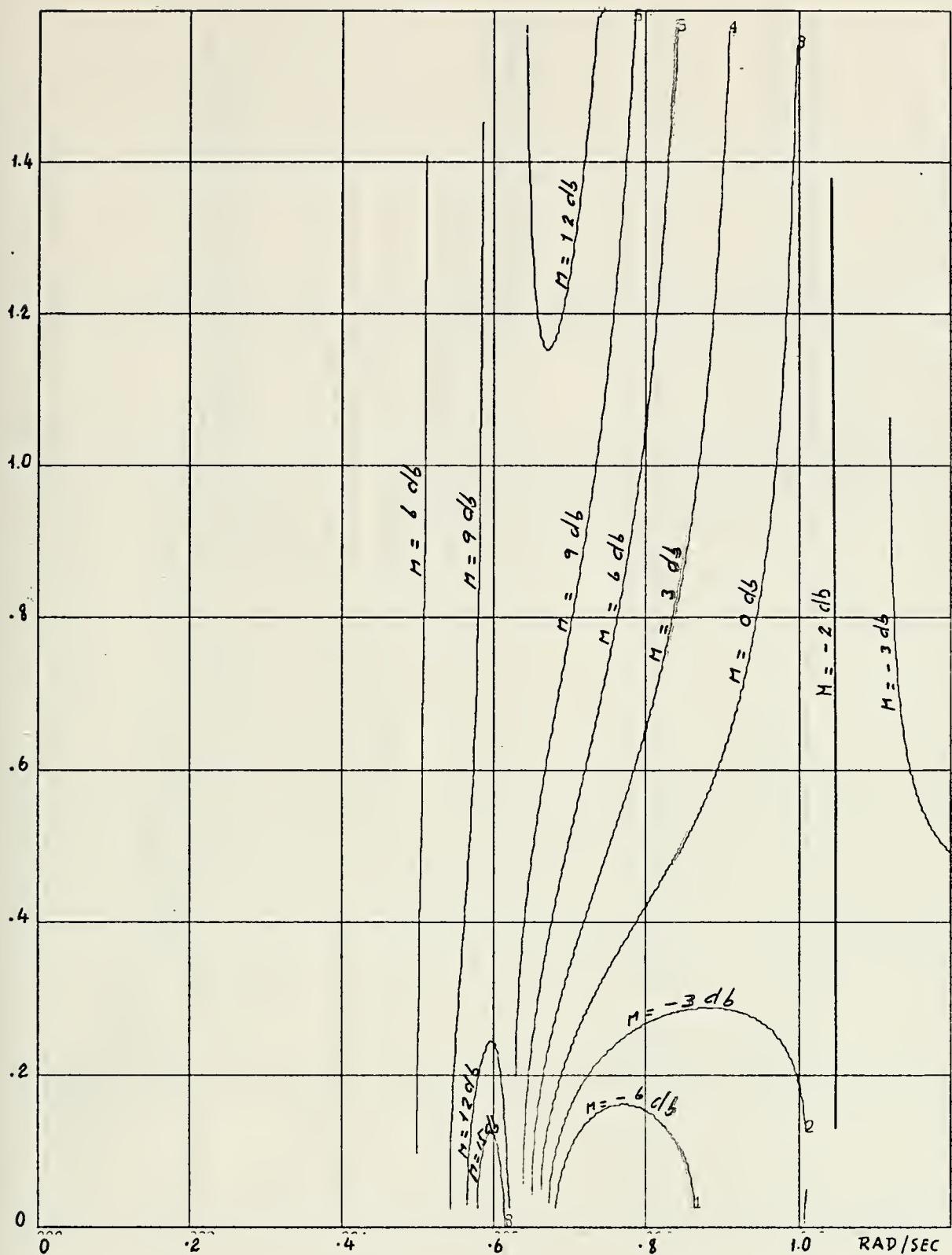


Figure 5.16a Frequency Parameter Plane for U-Tube Tank.
Plot of Tank Damping Factor vs Frequency
with $X = 3$, $\beta = 0.05$ and M as Curve Index.

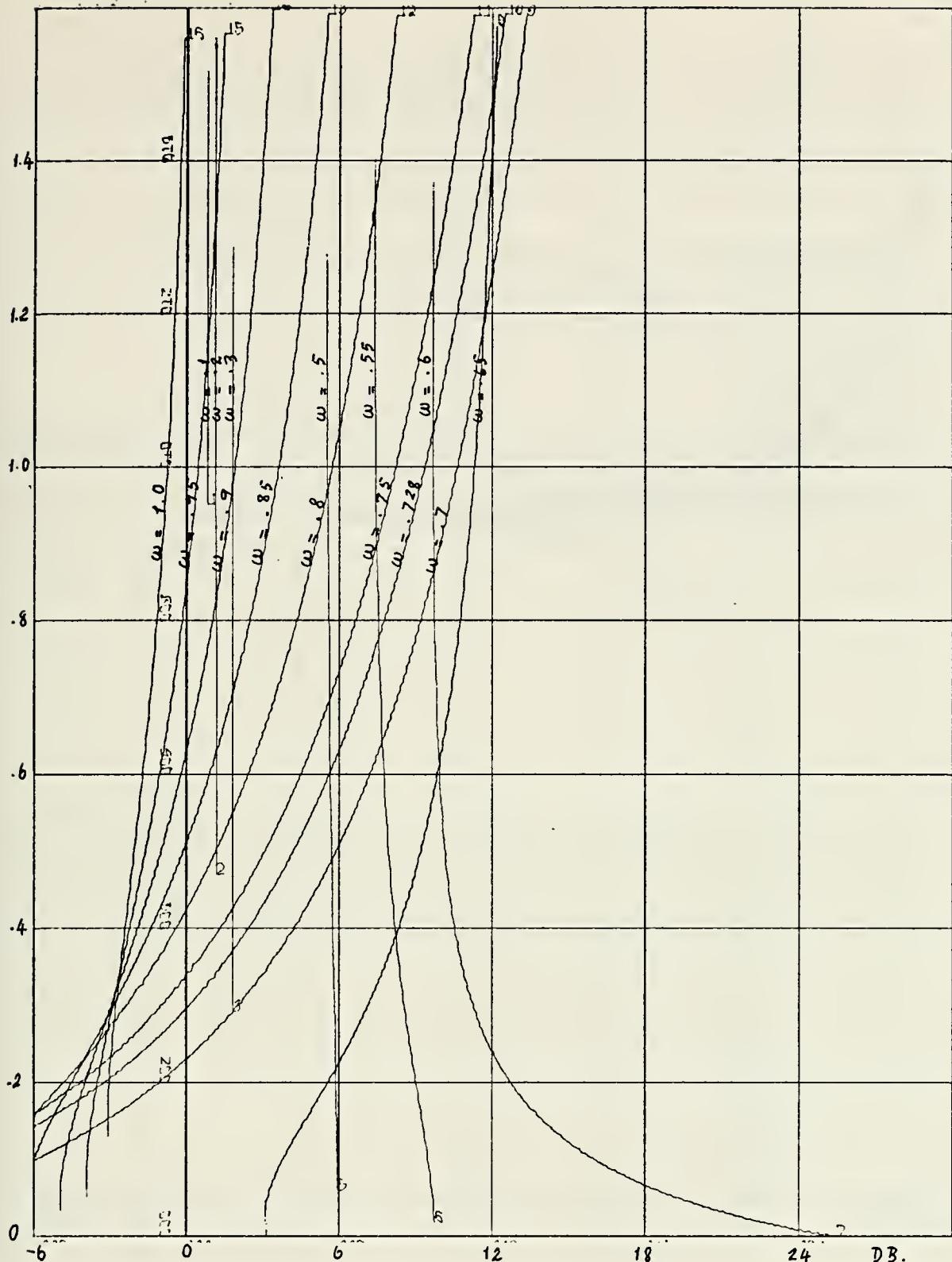


Figure 5.16b Frequency Parameter Plane for U-Tube Tank.
Plot of Tank Damping Factor vs Magnitude
with $X = 3$, $\beta = 0.05$ and ω as Curve Index.

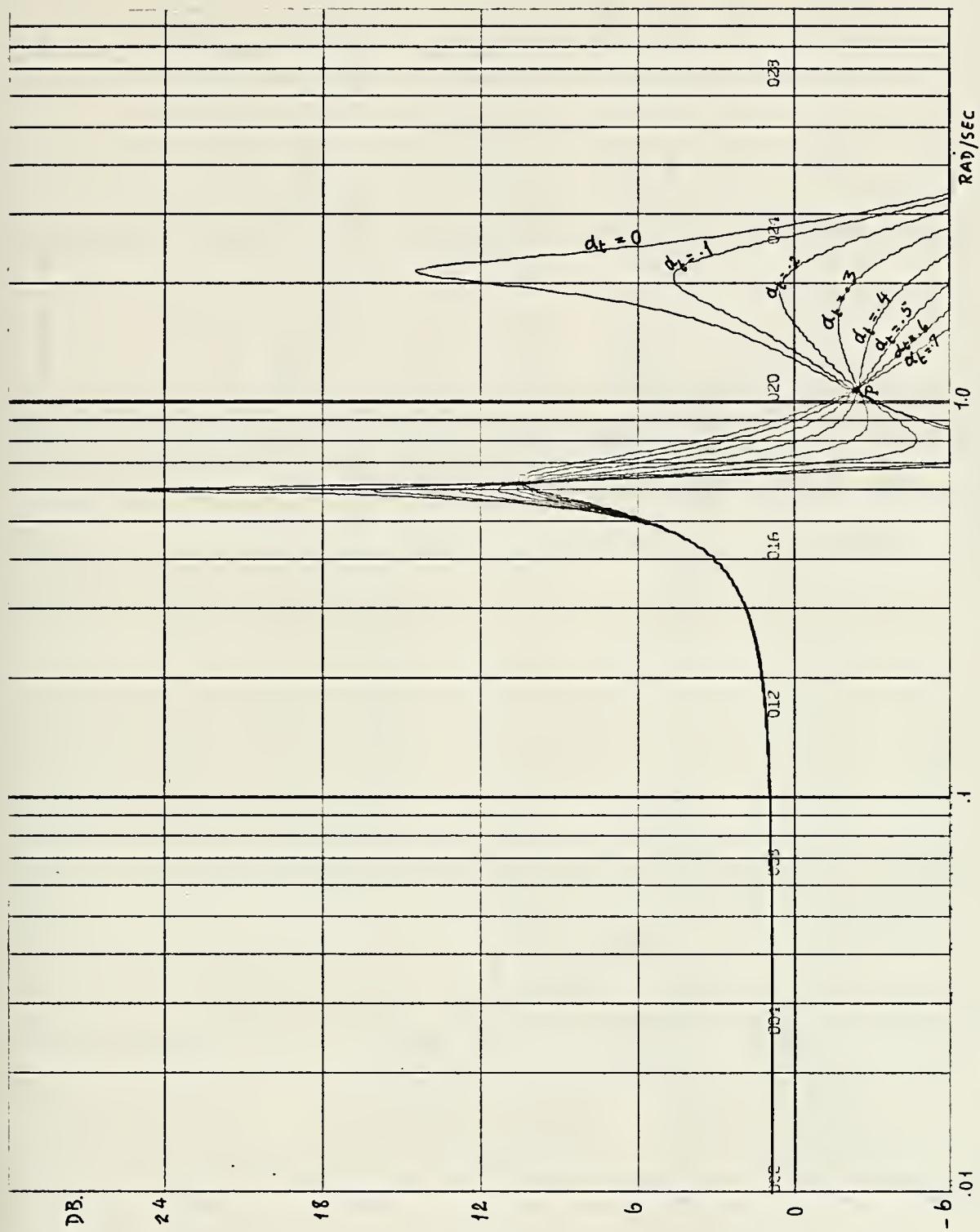


Figure 5.16c Frequency Parameter Plane for U-Tube Tank.
Bode Diagram, Plot of Tank Damping Factor
as Curve Index with $X = 3$ and $\beta = 0.05$.

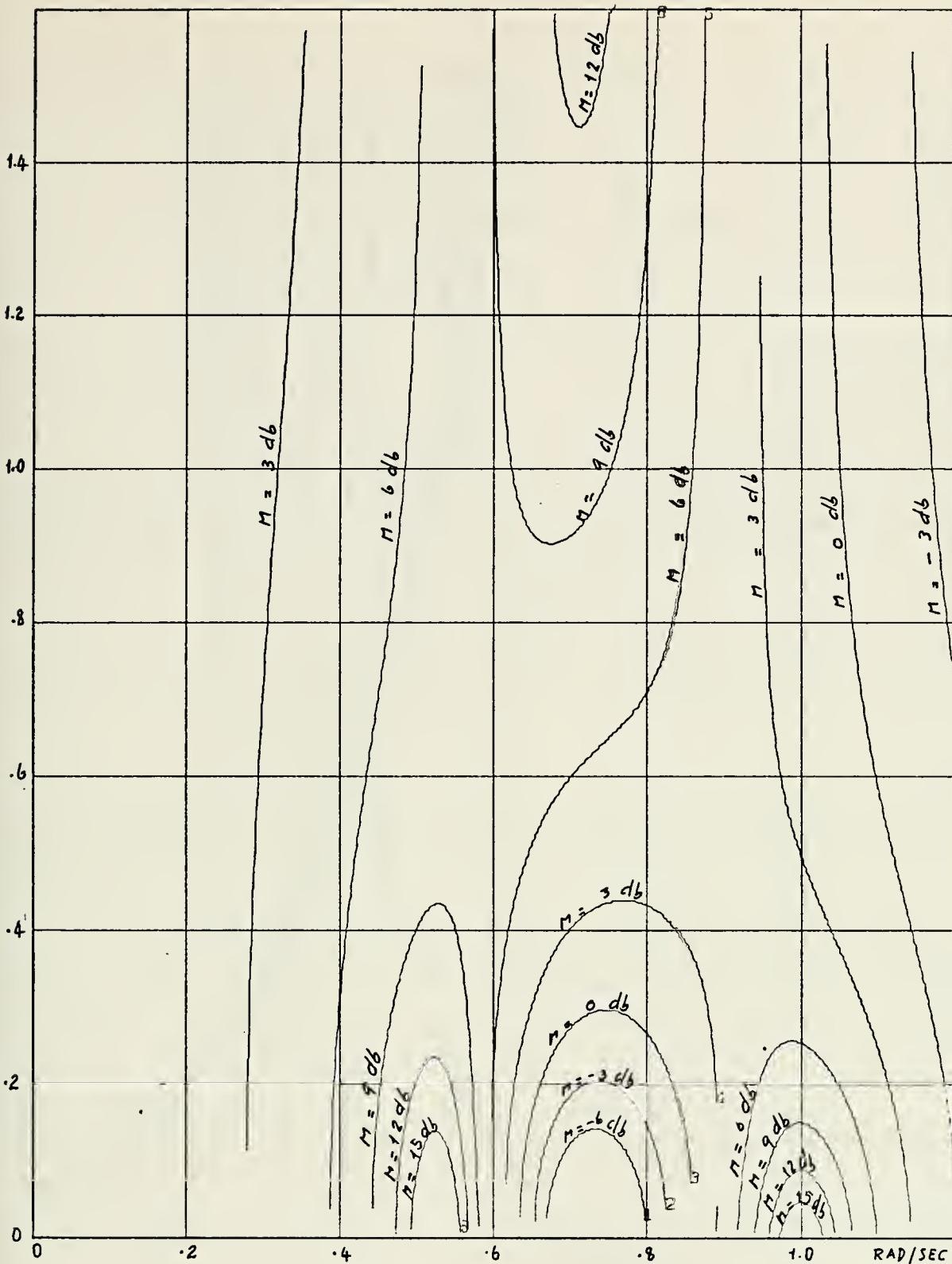


Figure 5.17a Frequency Parameter Plane for U-Tube Tank.
Plot of Tank Damping Factor vs Frequency
with $X = -1$, $\beta = 0.05$ and M as curve index.

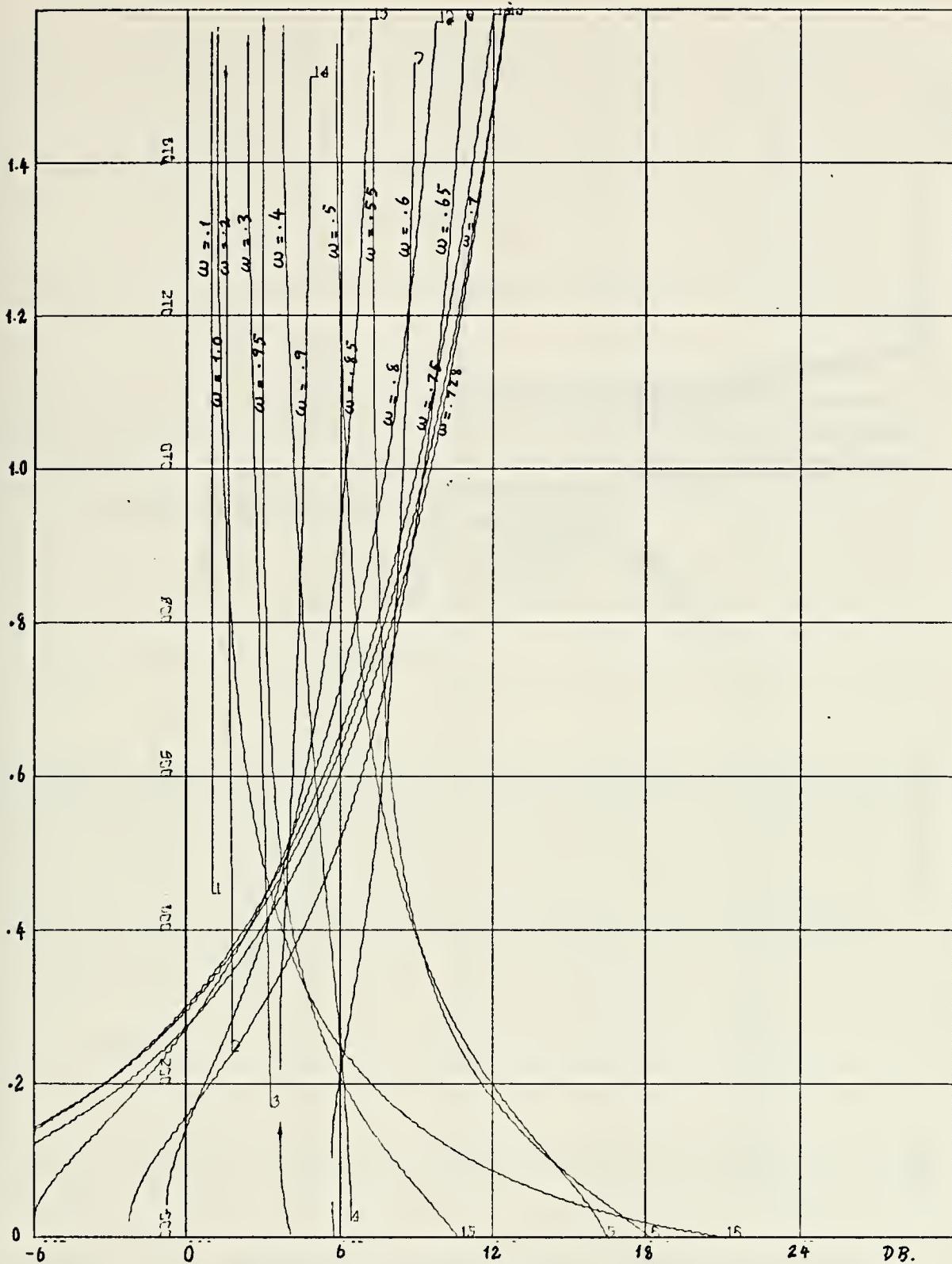
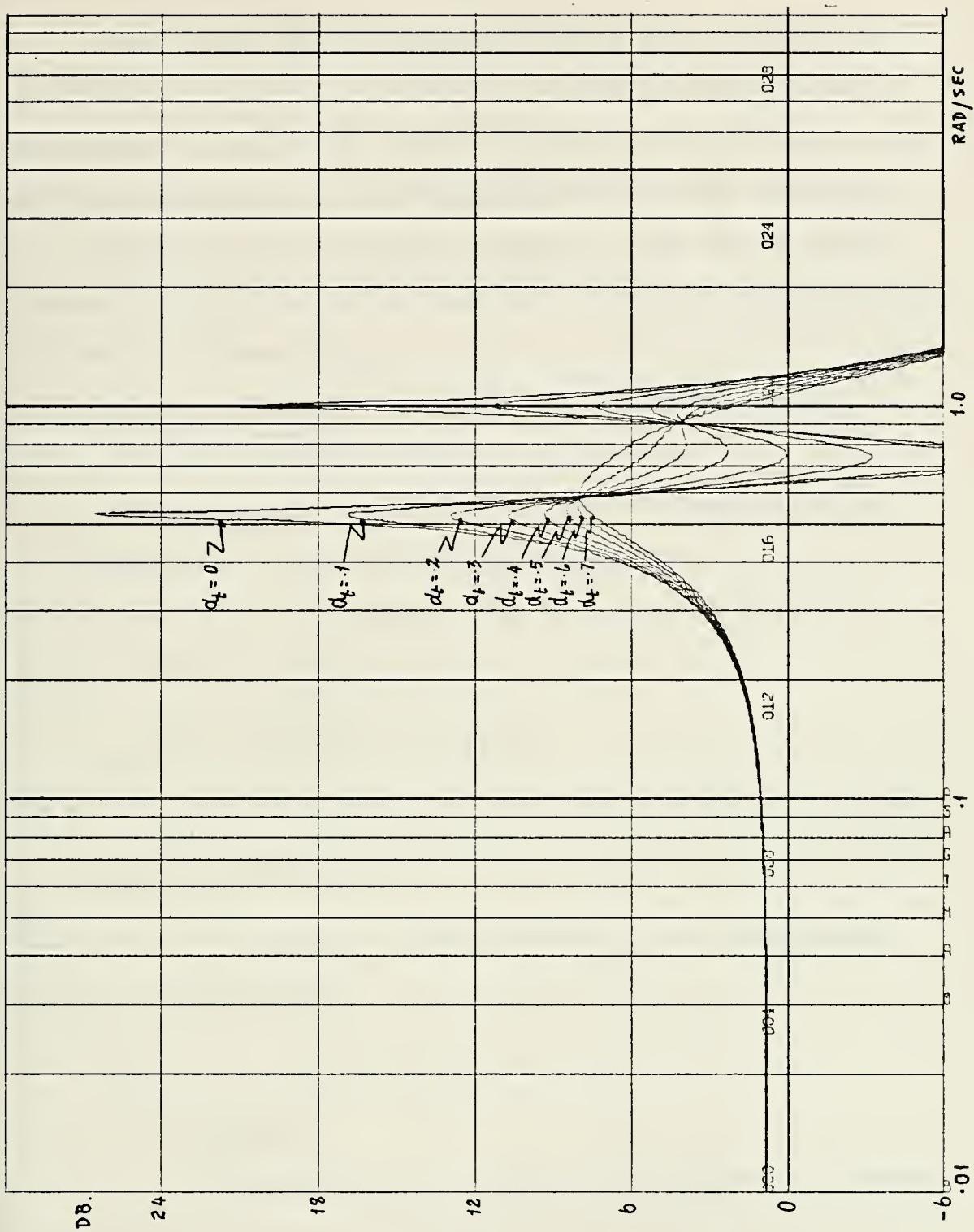


Figure 5.17b Frequency Parameter Plane for U-Tube Tank.
Plot of Tank Damping Factor vs Magnitude
with $X = -1$, $\beta = 0.05$ and ω as Curve Index.



**Figure 5.17c Frequency Parameter Plane for U-Tube Tank.
Bode Diagram, Plot of Tank Damping Factor
as Curve Index with $X = -1$ and $\beta = 0.05$.**

This is for the rectangular U-Tube tanks. With the equations as above, the values of cross-sectional area of the vertical leg of the tank, A_0 , the cross-sectional area of the channel, A_s , can be calculated and tank designed.

The depth of the water in tank, h , and the distance between the rolling center and the channel of the tank, D_s , can be calculated by:

$$X = 2 h + \frac{2 D_s A_s}{A_0}$$

In the case of $X = 3$, if the tank is located below rolling center, h should be less than 1.5.

E.g., if $X = 3$, $\beta=0.05$, the tank damping factor is 0.45. Then the tank size is:

$$A_0 R^2 = \frac{K_t}{2Pg} = \frac{\beta J_s}{2Pg} = 32.25 \text{ m}^4$$

if $R = 4$, the width of the tank is about 1 meter. Therefore

$$A_0 = 2.013 \text{ m}^2$$

and if $h = 1.3$ meters, the result of A_s is 0.4686 meter^2 and $D_s = 0.859 \text{ meter}$.

E. ACTIVE TANK STABILIZATION

1. Analysis

Activated-antiroll tanks are considered in detail. Particular attention is paid to the design of the tank control system. The equations of motion are the same as those of the ship with passive tank stabilizer, but must be modified to include the pump used in the activated tank. Active tank control can best be selected on the basis of minimizing the

response of the ship to an impulsive roll velocity, and it will be shown that the active tank is significantly better in reducing the roll of ship than the ordinary passive antiroll tank. The active tank stabilizer studied uses the U-Tube tank design.

Active tank analysis parallels that of passive tanks and/or the double pendulum. The tank system is activated by inserting a force (such as a pump system) into the second equation of passive tank system. The equation can then be written as:

$$\text{ship: } J_s \ddot{\phi} + B_s \dot{\phi} + K_s \phi + J_{st} \ddot{\theta} + K_{st} \theta = K_{ss} \psi$$

$$\text{tank: } J_{st} \ddot{\phi} + K_{st} \dot{\phi} + J_t \ddot{\theta} + B_t \dot{\theta} + K_t \theta = F_p$$

where F_p = the force due to pump system.

The effect of the roll sensing control on the force produced by the pump system can be expressed as:

$$F_p(s) = \frac{K_p(G_1 + G_2 s + G_3 s^2)\phi}{1 + \tau s}$$

where K_p = the gain of the control pump.

G_1 = roll angle sensitivity.

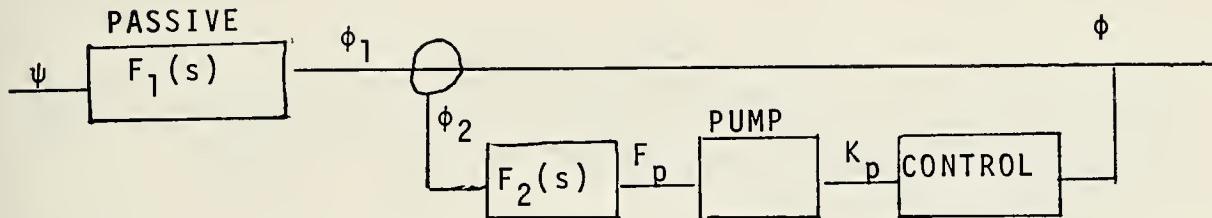
G_2 = roll rate sensitivity.

G_3 = roll acceleration sensitivity.

τ = time delay constant.

For convenience, we set $\tau=0$ in the computation, since the time delay is small.

The active tank system can be shown in block diagram form as:



where $F_1(s)$ = transfer function of the ship rolling due to wave slope alone which is the same as for the passive tank system.

$F_2(s)$ = transfer function of the ship rolling due to pump alone, which can be expressed as:

$$F_2(s) = \frac{\beta}{(1 - \frac{x^2 \beta}{\omega_s^2}) s^4 + (\alpha_s + \alpha_t) s^3 + (2\omega_n^2 + \alpha_t \alpha_s - 2x\beta) s^2 + \omega_n^2 (\alpha_t + \alpha_s) s + \omega_n^4 - \beta \omega_n^2}$$

$$= \frac{\phi_2}{F_p}$$

2. System Design

From the analysis in this section, the system performance can be studied using computer simulation. The pump coefficient factor K_p' ($K_p' = \frac{K_p}{J_t}$) is one variable parameter which may be called α . It is varied as the pitch ratio of the propeller of the pump, and can be assumed to be varied from zero to one and zero to minus one. For convenience the actual system may be designed with only positive variation and

the direction of the water motion between tanks is controlled by valves.

From the block diagram the system transmission function is:

$$\frac{\phi(s)}{\psi} = \frac{\omega_n^2 [s^2 + \alpha_t s + \omega_n^2]}{(1 - \frac{x^2 \beta}{\omega_n^2}) s^4 + (\alpha_t + \alpha_s) s^3 + (2\omega_n^2 + \alpha_t \alpha_s - 2x\beta - K_p' G_3 \beta) s^2 + (\omega_n^2 \alpha_t + \omega_n^2 \alpha_s - K_p' G_2) s + \omega_n^4 - \beta \omega_n^2 - K_p' G_1 \beta}$$

where $F_p(s) = K_p(G_1 + G_2 s + G_3 s^2) \phi$.

Define $X = 3$

$\beta = 0.05$

$\alpha_t = 0.4$

and $K_p' = \alpha$

These values for a passive tank gave a peak resonance of minimum magnitude at about 10 db at 0.6 rad/sec and is 1 db at the lower frequencies. Using these values the equation becomes:

$$(s) = \frac{0.53023s^2 + 0.212092s + 0.28115}{0.1513s^4 + 0.4465s^3 + (0.7791 - 0.05\alpha G_3)s^2 + (0.237 - 0.05\alpha G_2)s + 0.25464 - 0.05\alpha G_1}$$

Figure 5.18 shows the Bode diagram for $G_1 = 1$, $G_2 = G_3 = 0$. If α is more positive, the frequency and the magnitude of the resonance peak are slightly decreased. At the low frequencies, the magnitude is increased. If α is more negative, the result is reversed. To obtain the minimum

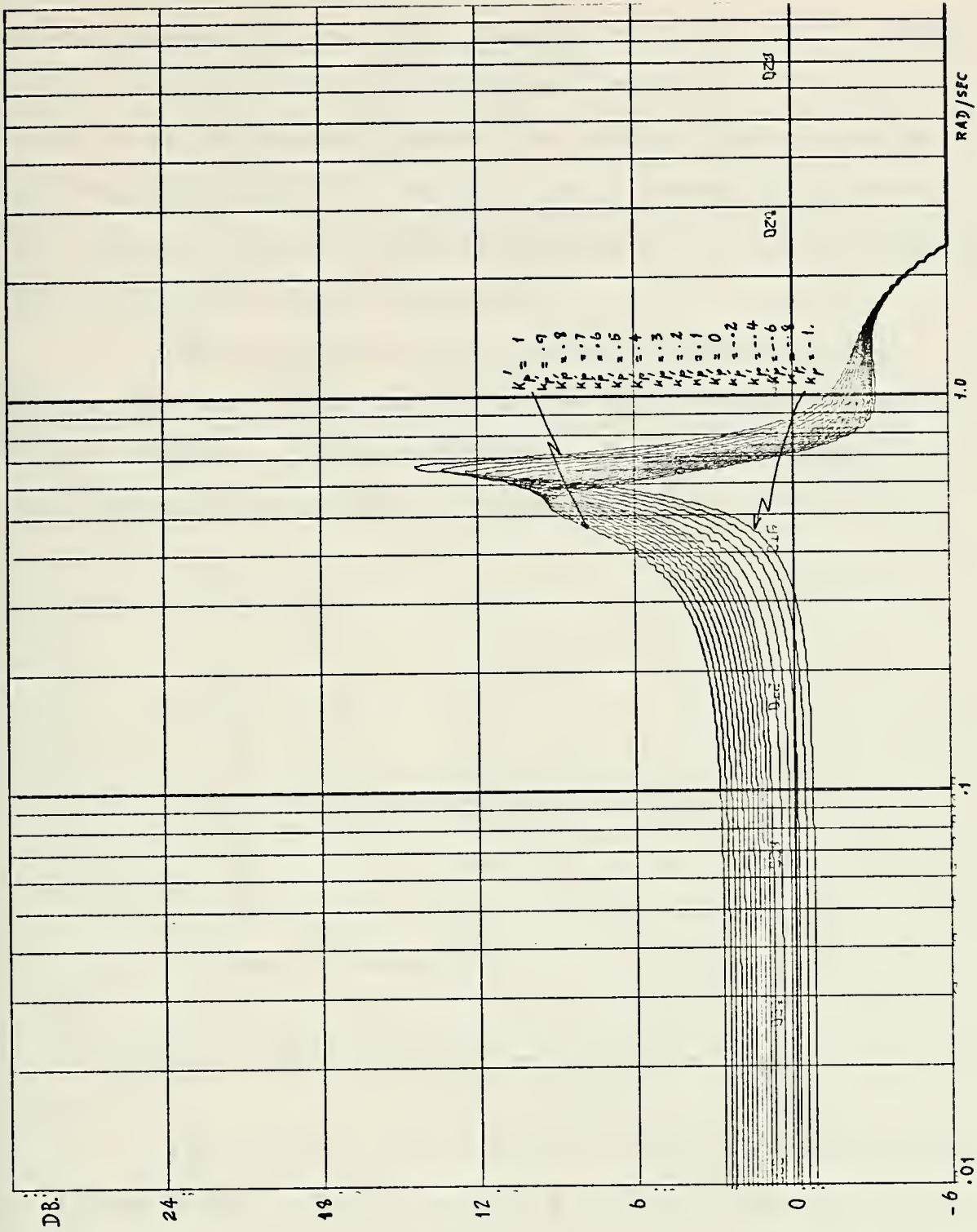


Figure 5.18 Frequency Parameter Plane for Activated Tank.
Bode Diagram, Plot of Pump Coefficient Factor
as Curve Index ($G_1=1$, $G_2=G_3=0$) and $X=3$.

magnitude gain, at the lower frequencies from 0 to 0.6 rad/sec α is -1, for the higher frequencies α must be +1. So, the pitch ratio of the propeller of the pump may be kept constant and the direction of the water motion reversed by switching the valves. Then the maximum magnitude is 7 db at 0.6 rad/sec and -1 db at the lower frequencies.

The characteristics of the active device (pump, valves, sensor) can be treated as the variation of the value of the parameter α versus frequency. This value is required to switch at the resonant frequency as shown in Figure 5.19.

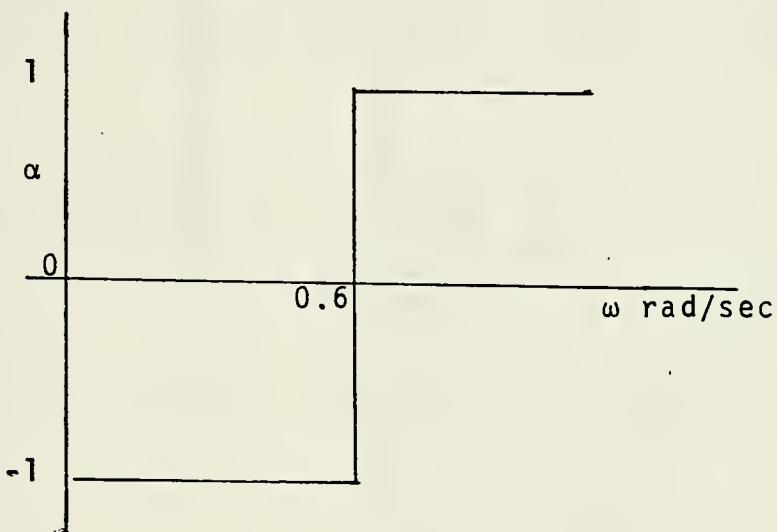


Figure 5.19 The variation of control device.
 $(X=3, G_1=1, G_2=G_3=0)$

For $G_1=0.5$, $G_2=1.25$ and $G_3=2.4$ the results are shown as Figure 5.20a, b and c. Figure 5.20a is a plot of α versus frequency, and magnitude is the curve index. When α is more negative the magnitude is seen to be smaller. If $\alpha=1$ the maximum peak magnitude is obtained at the frequency of about

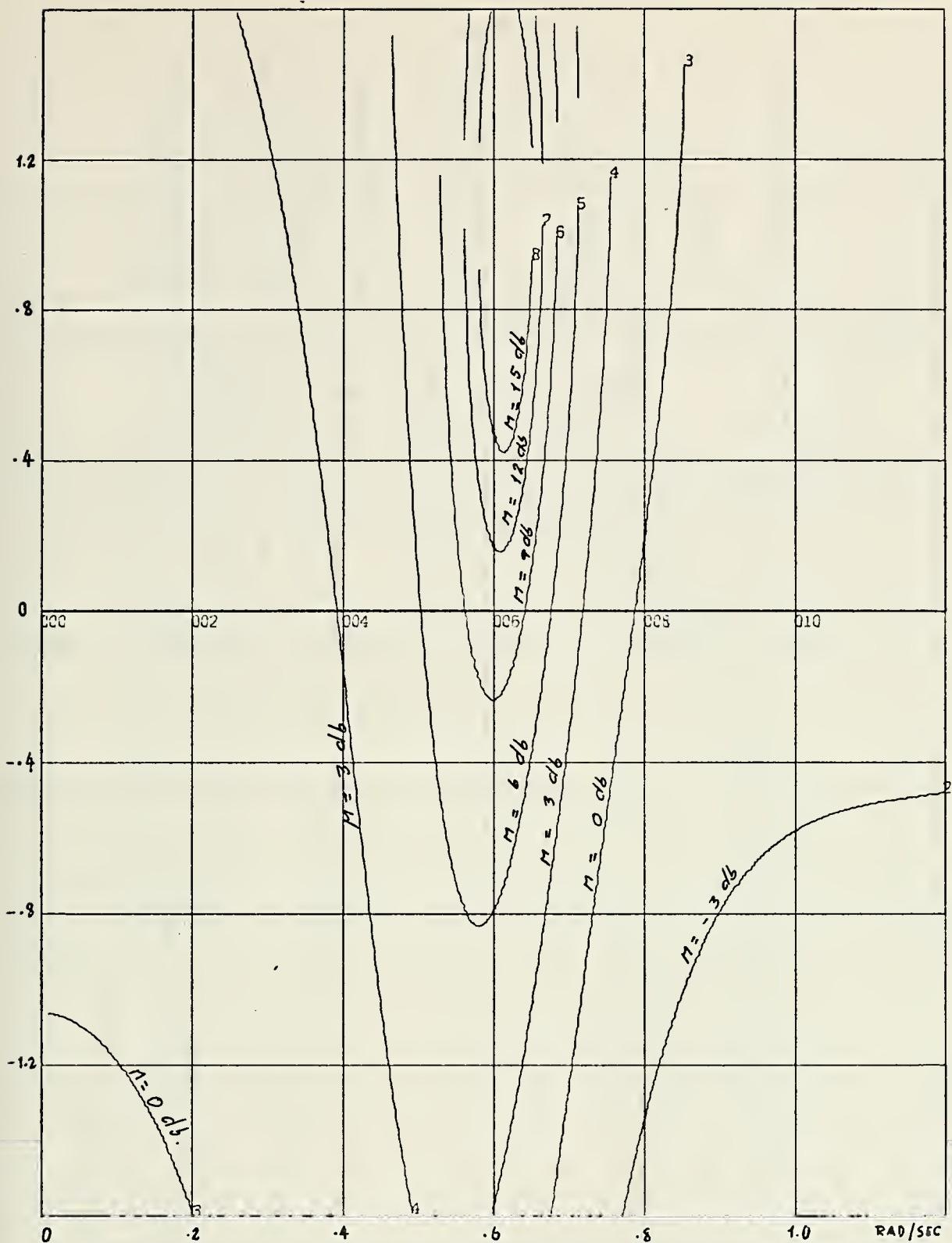


Figure 5.20a Frequency Parameter Plane for Activated Tank.
 Plot of Pump Coefficient Factor vs Frequency
 with M as Curve Index ($G_1=0.5$, $G_2=1.25$,
 $G_3=2.4$) and $X=3$.

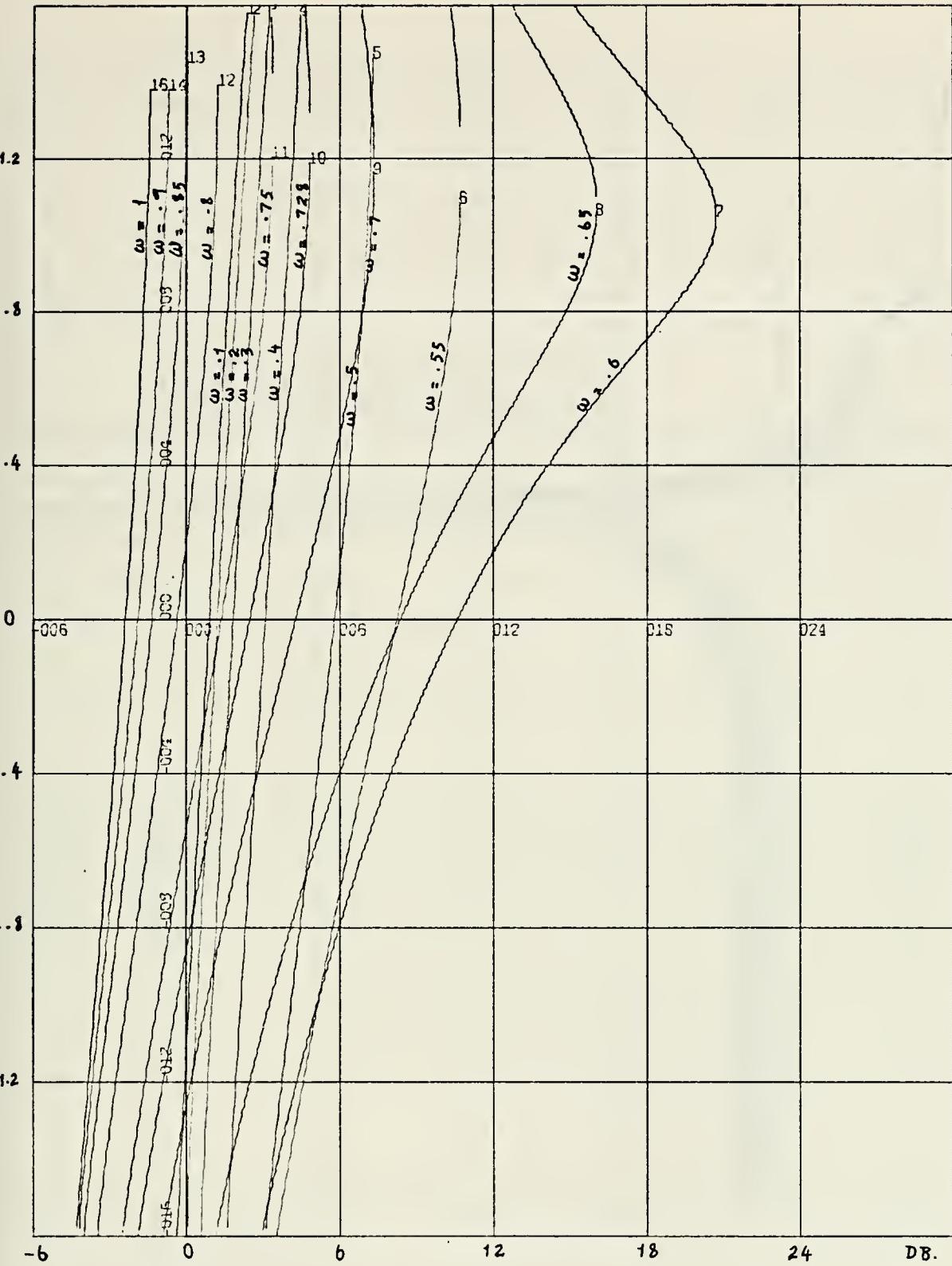


Figure 5.20b Frequency Parameter Plane for Activated Tank. Plot of Pump Coefficient Factor vs Magnitude with ω as Curve Index ($G_1=0.5$, $G_2=1.25$, $G_3=2.4$) and $X=3$.

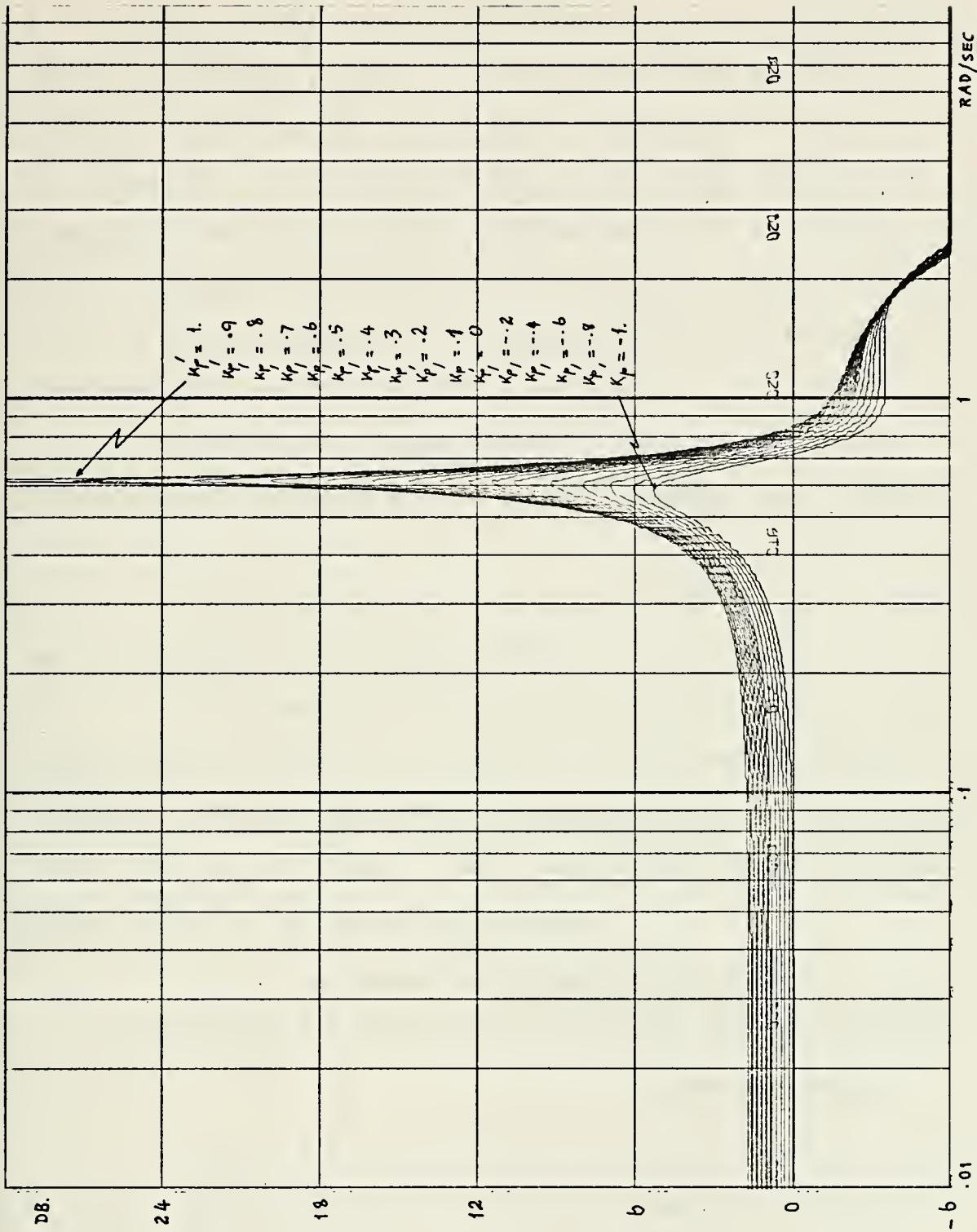


Figure 5.20c Frequency Parameter Plane for Activated Tank.
Bode Diagram, Plot of Pump Coefficient Factor
as Curve Index ($G_1=0.5$, $G_2=1.25$, $G_3=2.4$) and
 $X=3$.

0.6 rad/sec. These results correspond to Figure 5.20b which shows the frequency curve. At the same frequency of 0.6 rad/sec, both figures show maximum magnitude for all values of α from -1 to +1. Figure 5.20c is the Bode diagram and shows that there is no phase lag between ship and active tank, and if α is more negative the magnitude is smaller. So, at $\alpha=-1$, it produces the smallest magnitude at all frequencies, with the peak equal 5 db at 0.6 rad/sec.

This result seems to be the best since all the parameters of the pump system (pitch, sensing gain control, and valves) are fixed.

From both results as mentioned above, the system can be designed as a linear device.

If there is no limit on the height at which the tank can be located, it is possible to choose the height, so that $X=0$. If this is done, by inspection the value of $\beta=0.05$ and $\alpha_t=0.3$ are obtained. Substitute these values into the equation of the system which becomes:

$$(s) = \frac{0.53023s^2 + 0.159069s + 0.28115}{s^4 + 0.3465s^3 + (1.07436 - 0.05K_p G_3)(s^2 + (0.18373 - 0.05K_p G_2)s + 0.25464 - 0.05K_p G_1)}$$

Figure 5.21a and b show the results with $G_1=1$ and $G_2=G_3=0$. Figure 5.21a is a plot with magnitude as curve index; point Q is the maximum magnitude with $\alpha=-0.65$ at a frequency of 0.728 rad/sec. If $\alpha=0.15$, the maximum magnitude of 9 db is obtained which is the minimum peak. Figure 5.21b

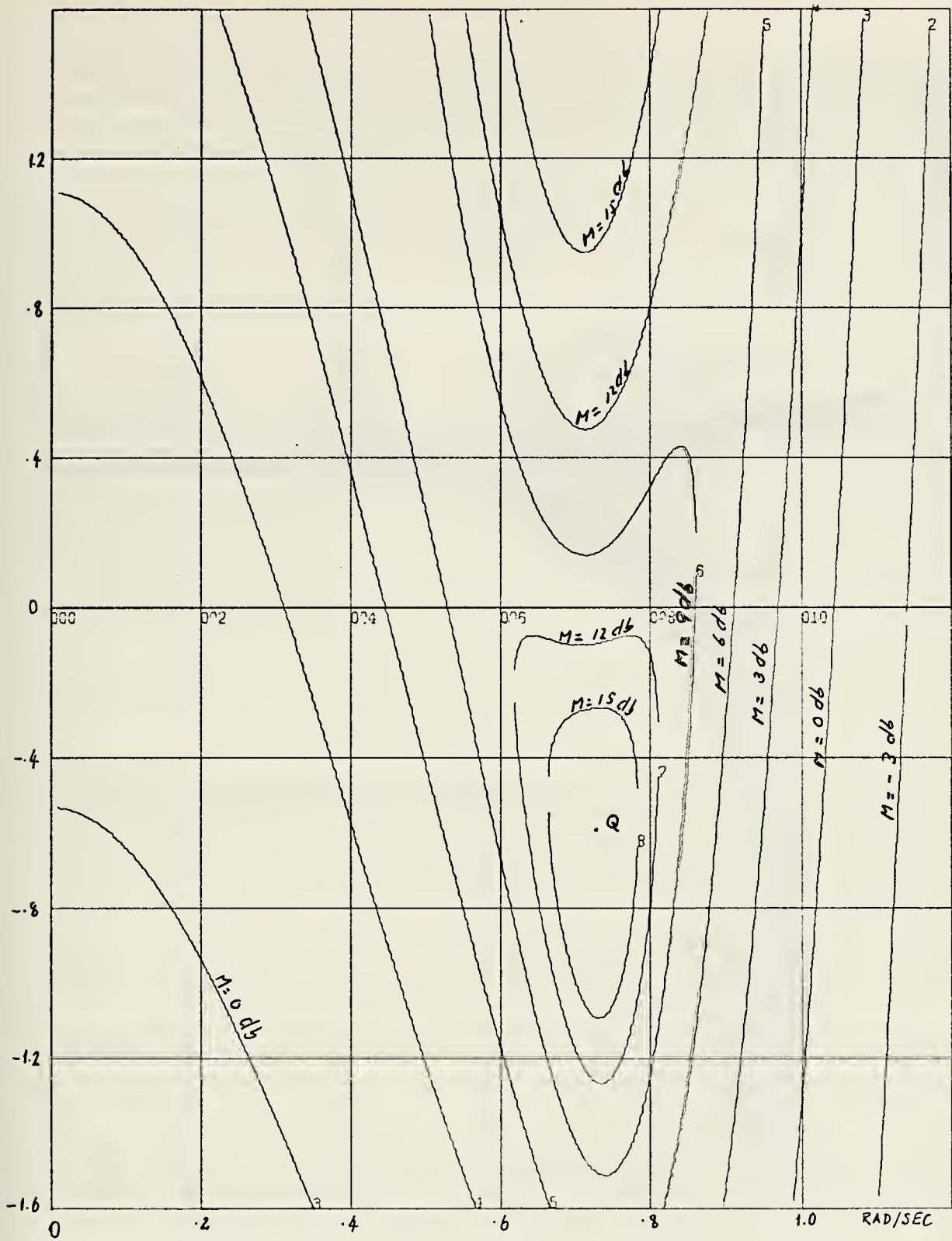


Figure 5.21a Frequency Parameter Plane for Activated Tank.
Plot of Pump Coefficient Factor vs Frequency
with M as Curve Index ($G_1=1$, $G_2=G_3=0$) and $X=0$.

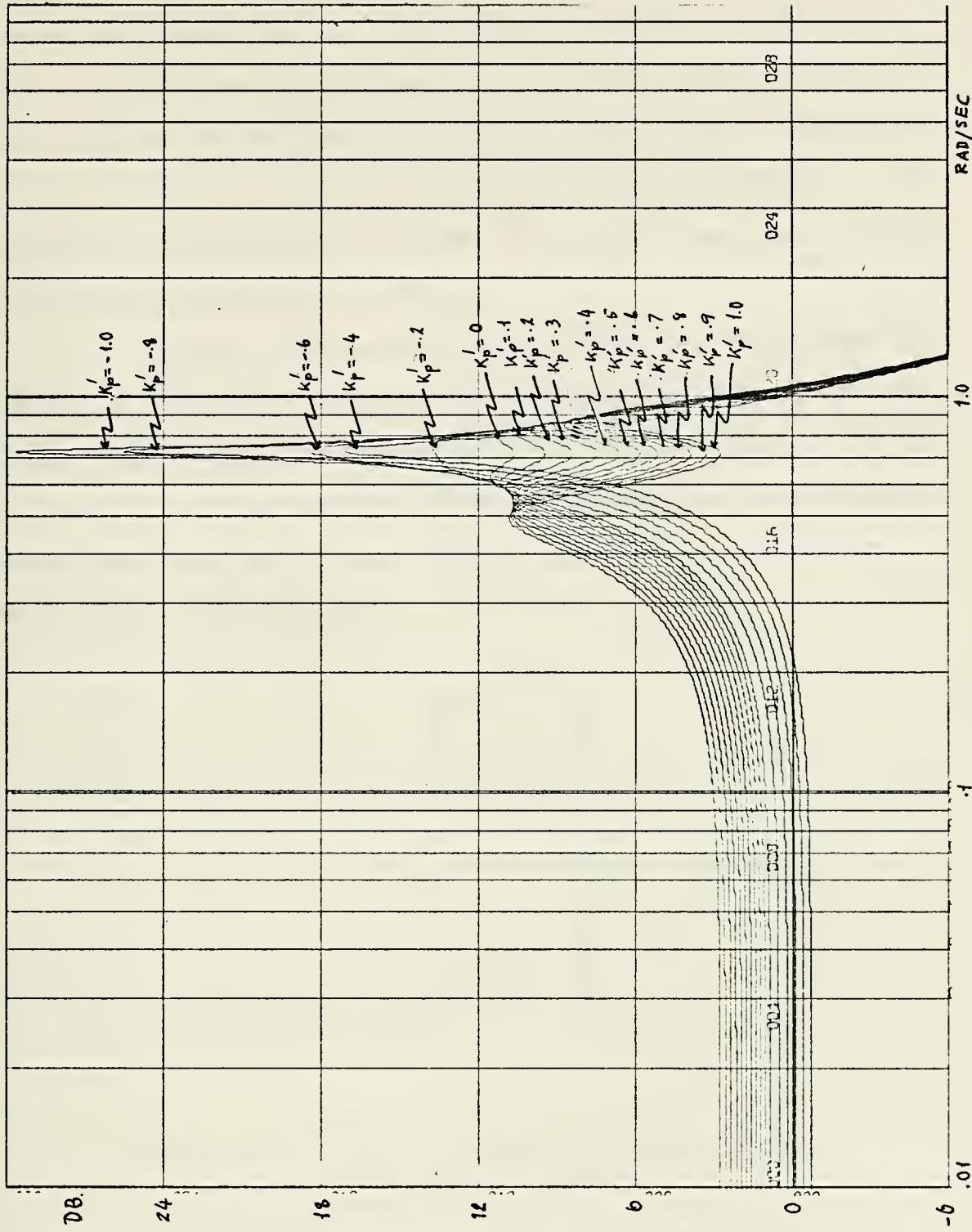


Figure 5.21b Frequency Parameter Plane for Activated Tank.
Bode Diagram, Plot of Pump Coefficient Factor
as Curve Index ($G_1=1$, $G_2=G_3=0$) and $X=0$.

shows the Bode diagram. To obtain the minimum magnitude, in the frequency region between 0.6 and 0.85 rad/sec the value of α must be maintained at +1 and for the other frequencies, at -1. There are two magnitude peaks of approximately 7 db at the intersection of $\alpha=+1$ and $\alpha=-1$. For the lower frequencies the magnitude is -1 db.

From the above result the active device parameter value, α , can be plotted versus frequency as in Figure 5.22. This figure shows two values of α at the frequencies between 0.6 and 0.85, the straight line corresponds to the minimum magnitude, and the curved line corresponds to a constant magnitude in that region.

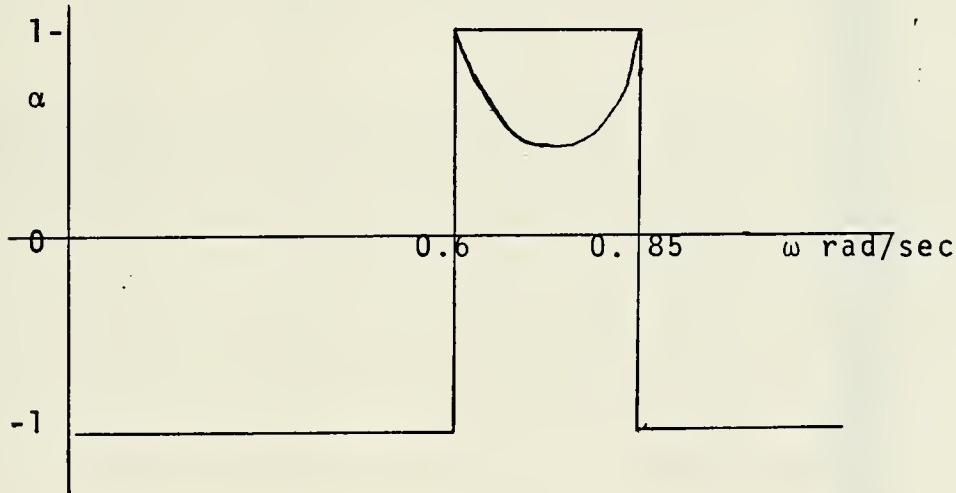


Figure 5.22 The Variation of Active Device
($X=0$, $G_1=1$, $G_2=G_3=0$)

Figure 5.23 shows the Bode diagram of the system for $G_1=0.5$, $G_2=1.25$, and $G_3=2.4$. For minimum magnitude with the control device, the required variation in α is shown in Figure 5.24.

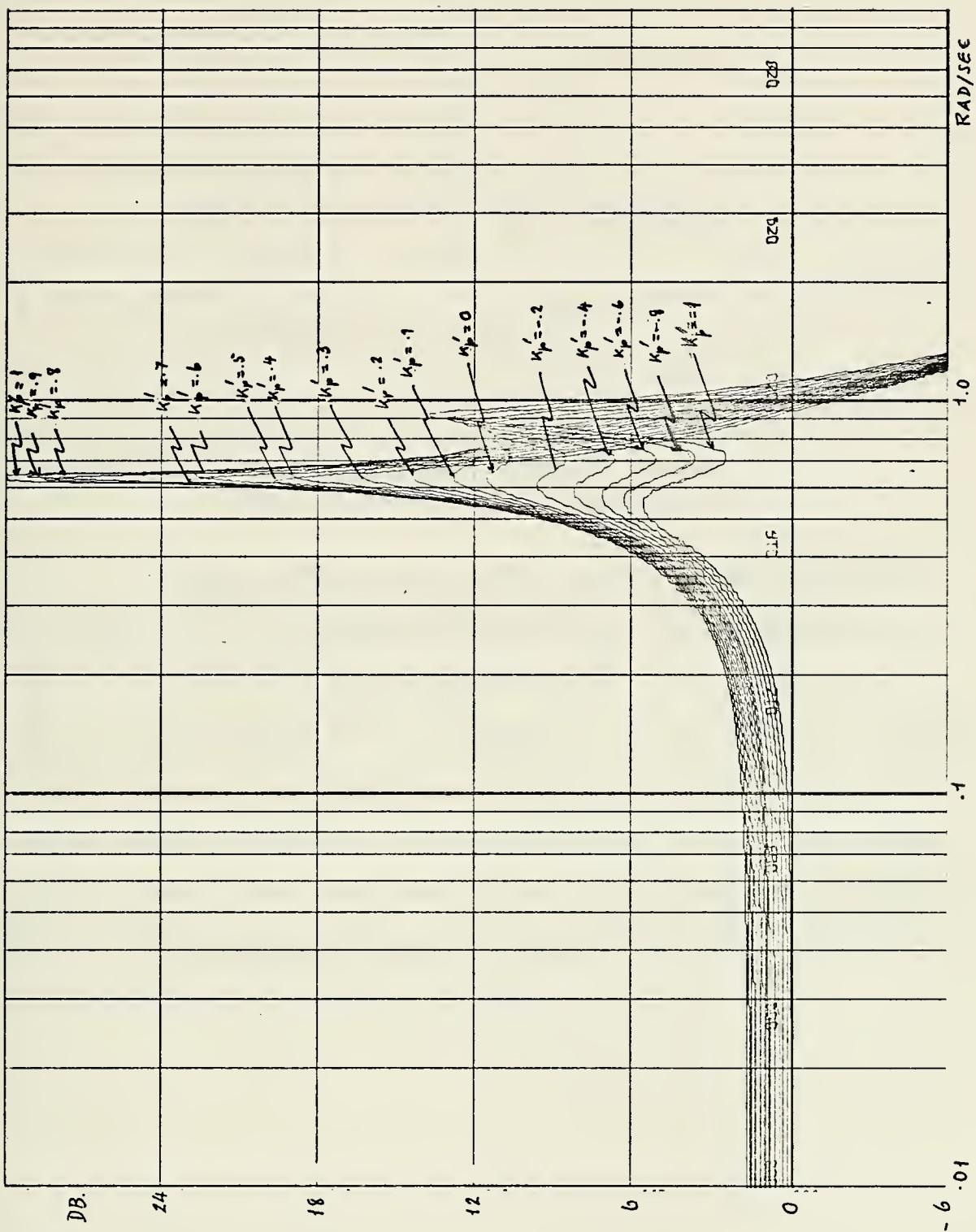


Figure 5.23 Frequency Parameter Plane for Activated Tank.
Bode Diagram, Plot of Pump Coefficient Factor
as Curve Index ($G_1=0.5$, $G_2=1.25$, $G_3=2.4$) and
 $X=0$.

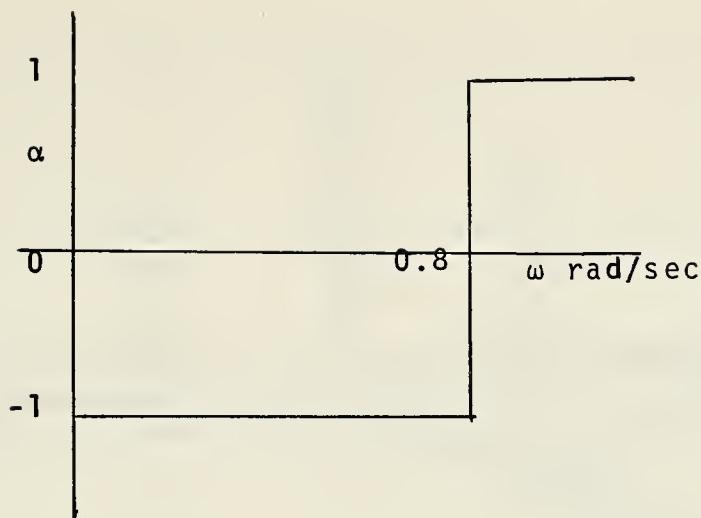


Figure 5.24 The Variation of Control Device
 $(X=0, G_1=0.5, G_2=1.25 G_3=2.4)$

Figure 5.25, which is the Bode diagram, compares the results for an undamped ship and for a ship damped by a tank. All have only one resonance peak. The magnitude of roll in the undamped ship at resonance is the highest, damping clearly reduces the amplitude. However, in the passive tank system the magnitude at lower frequencies is slightly higher, and the curves show that the U-Tube tank is more advantageous than the free-surface tank. The active tank system gave the most advantageous behavior as shown.

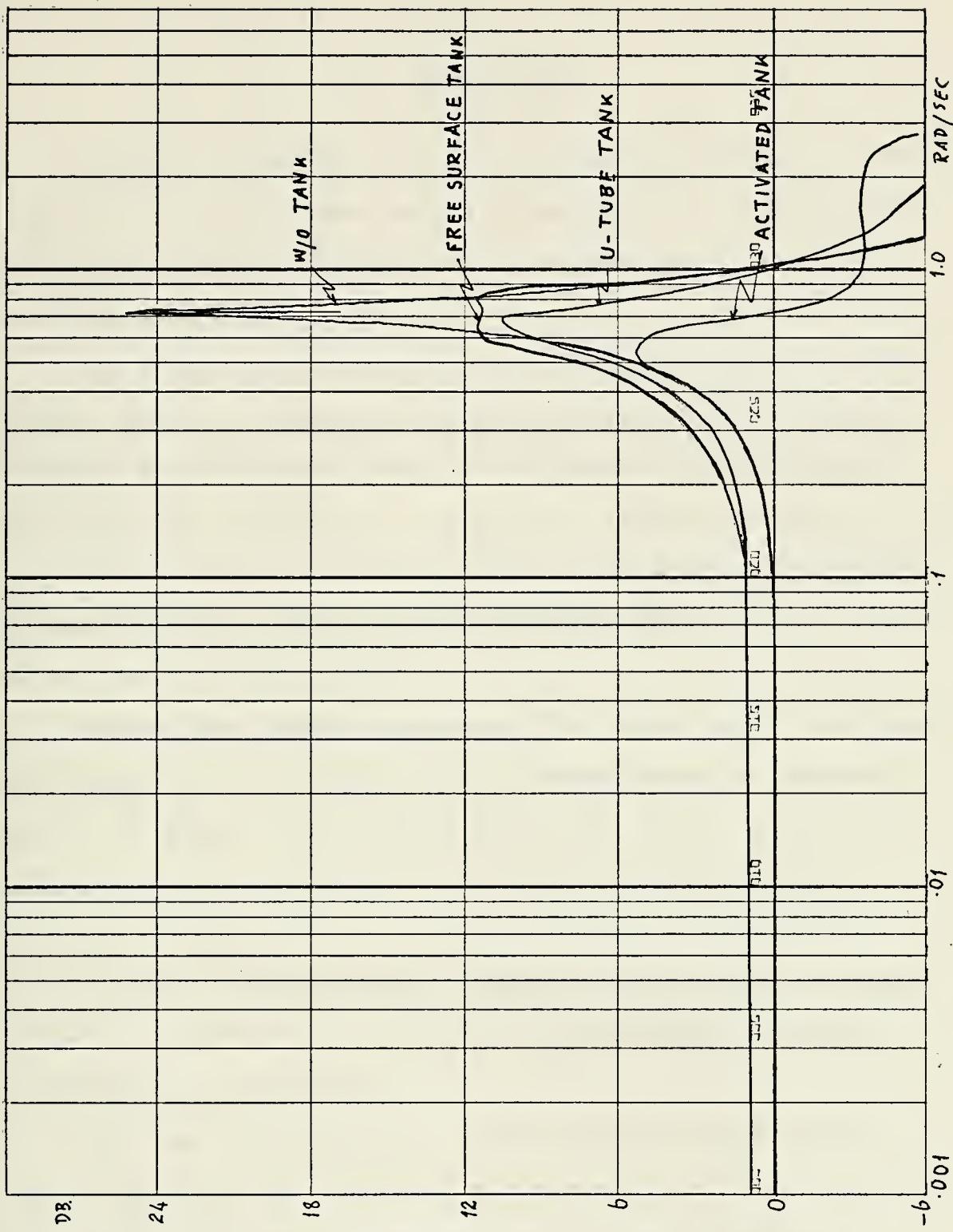


Figure 5.25 Compare the Results in Magnitude Gain.

VI. CONCLUSION

Analysis techniques can be employed effectively in system design particularly when an exact analytical solution is not available and where a trial and error method is the only solution possible.

The frequency parameter plane techniques have been used in this thesis to analyze and design a system. The system elements which may be linear or nonlinear were represented by adjustable parameters. To select a stable system, the root parameter plane was used and further mapped onto the frequency parameter plane. The goal was to obtain the best set of parameters for a stable system.

Rather than adjust parameters for every trial, one graph was made and the parameter value corresponding to the best curve was chosen. This appreciably reduced the number of trials and the computation time in the overall design.

In the system which is presented the tank stabilizer design might not be optimal, however further effort on parameter adjustments by iteration of the described technique would improve the design.

The tank stabilizers of a ship were designed in this thesis by using the frequency parameter plane technique, they were very successful and effective. If the relationship between the designed parameters and tank size are true, the result of designing by using the method would be feasible and

believable. In passive tank design, the tank size depends on the free surface factor. The tank damping factor depends on the skin of the material of the tank, which would be tested to obtain its value from the analysis. The value of X in U-Tube tank would have to be chosen within the range which maintains ship stability.

In the activated tank design, the gain control constants were varied by trial and error to adjust the damping. Physically only one parameter can be varied, the pitch radio of the propeller of the pump.

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The four dimensional parameter space relating magnitude, frequency, and the two parameters may be represented by seven two dimensional projections of the parametric curves. Computer programs are developed for the computation, and graphs of results are obtained. Interpretations of the graphs can be used for design of systems such as tank stabilizers.

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